ADDITION OF FRACTIONS.

129. Addition of Fractions is the operation of finding the sum of two or more fractions.

1. Henry gave $\frac{1}{2}$ of an apple to John, and $\frac{1}{2}$ of an apple to Charles: how many half apples did he give away?

SOLUTION. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$.

2. James bought $\frac{1}{4}$ of a bushel of wheat in the morning, $\frac{1}{4}$ in the afternoon, and $\frac{1}{4}$ in the evening: what part of a bushel did he buy during the day?

SOLUTION. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

3. How many fifths in $\frac{1}{5} + \frac{2}{5} + \frac{3}{5}$? *Ans.* $\frac{6}{5} = 1\frac{1}{5}$.

To add fractions they must have common denominators.

4. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

ILLUSTRATION.

$$\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{40}{60} + \frac{45}{60} + \frac{48}{60}.$$

$$\frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = 2\frac{13}{60}.$$

EXPLANATION.—Here we reduce the fractions to equivalent fractions having a common denominator, add their numerators and place their sum over the common denominator; then, because the sum is an improper fraction, we reduce it to a mixed number.

From the above explanation, we have the following

RULE.

I. When the fractions have the same denominator, add the numerators, and place their sum over the common denominator. If the result is an improper fraction, reduce it to a whole or mixed number.

II. When they have not the same denominator, reduce them to a common denominator, and add as before.

EXAMPLES FOR ORAL WORK.

1. Add $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{5}{4}$.

2. What is the sum of $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$, and $\frac{6}{5}$?

3. If Jane pays $\frac{1}{5}$ of a dollar for paper, $\frac{1}{10}$ of a dollar for a pencil, and $\frac{3}{10}$ for a bottle of ink, how much does she pay for the whole?

4. Henry spends $\frac{1}{6}$ of a dollar at one time, \$$1\frac{5}{6}$ at another, \$$2\frac{1}{6}$ at another; how much does he spend?

5. How many twenty-fourths are there in $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$? How many in $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$? How many in $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{2}{3}$.

6. Add $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{10}$.

7. Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{2}{9}$.

8. Add $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{2}{16}$.

9. Add 9, $\frac{2}{3}$, $\frac{1}{15}$, $\frac{5}{6}$, and $\frac{2}{3}$.

10. Add 12, $\frac{3}{10}$, $\frac{6}{20}$, and $\frac{7}{30}$.

11. Add $\frac{4}{9}$, $\frac{17}{90}$, $\frac{13}{30}$, and 15.

EXAMPLES FOR WRITTEN WORK.

Add the following groups of fractions:

1. $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{16}$.

2. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

3. $\frac{7}{25}$, $\frac{11}{30}$, and $\frac{8}{15}$.

4. $\frac{2}{15}$, $\frac{5}{16}$, and $\frac{11}{20}$.

5. $\frac{7}{9}$, $\frac{11}{15}$, and $\frac{11}{12}$.

6. $\frac{3}{5}$, $\frac{2}{9}$, $\frac{2}{3}$, and $\frac{7}{15}$.

7. $\frac{2}{27}$, $\frac{3}{54}$, $\frac{7}{9}$, and $\frac{5}{18}$.

8. $\frac{5}{32}$, $\frac{3}{28}$, and $\frac{11}{20}$.

9. $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.

10. $\frac{1}{15}$, $\frac{5}{21}$, and $\frac{7}{18}$.

To add mixed numbers, we add the integral parts and the fractional parts separately, and then find the sum of the results; or, reduce them to improper fractions and add by the last rule.

Add the following groups of fractions:

11. $2\frac{1}{2}$, $3\frac{3}{4}$, and $4\frac{7}{8}$.

12. $3\frac{3}{7}$, $4\frac{11}{14}$, and $5\frac{23}{28}$.

13. $\frac{7}{18}$, $2\frac{1}{4}$, 8, and $8\frac{5}{6}$.

14. $\frac{1}{3}$, $2\frac{1}{5}$, $7\frac{1}{9}$, and $4\frac{5}{12}$.

15. $4\frac{1}{9}$, $7\frac{1}{5}$, and $3\frac{11}{45}$.

16. $2\frac{5}{7}$, $\frac{3}{7}$, $4\frac{9}{14}$, and 8.

17. $3\frac{1}{5}$, $2\frac{3}{10}$, $5\frac{11}{15}$, and $6\frac{7}{20}$.

18. $13\frac{1}{9}$, $5\frac{3}{10}$, and $\frac{11}{45}$.

19. $3\frac{2}{3}$, $4\frac{2}{5}$, and $\frac{1}{15}$.

20. $2\frac{1}{8}$, $7\frac{4}{5}$, and $2\frac{3}{10}$.

PRACTICAL PROBLEMS.

1. A. bought 3 pieces of cloth; the first contained $39\frac{1}{2}$ *yds.*, the second $38\frac{1}{4}$ *yds.*, and the third $40\frac{7}{8}$ *yds.*; how many yards in all? *Ans.* $118\frac{5}{8}$.

2. John bought a sled for $\$3\frac{1}{2}$, a pair of skates for $\$1\frac{7}{8}$, and a cap for $\$4\frac{3}{16}$; what did they all cost him? *Ans.* $\$9\frac{9}{16}$.

3. A farmer bought a farm, of which $27\frac{1}{4}$ *acres* were woodland, $49\frac{3}{8}$ *acres* pasture, $17\frac{4}{5}$ *acres* ploughland, and $36\frac{1}{10}$ *acres* meadow; what was the content of the farm? *Ans.* $130\frac{21}{40}$ *acres.*

4. A man spent at a store $\$7\frac{3}{8}$ for a barrel of flour, $\$6\frac{1}{6}$ for sugar, $\$5\frac{4}{9}$ for tea, and had $\$2\frac{1}{2}$ remaining; how much had he at first?

5. A merchant sold cloth for $\$37\frac{1}{2}$, ribbons for $\$2\frac{7}{8}$, thread for $\$1\frac{1}{16}$, and pins for $\$\frac{7}{8}$; how much did they all amount to?

6. Find the sum of $7\frac{1}{2}$ *lbs.*, $4\frac{1}{4}$ *lbs.*, $3\frac{7}{8}$ *lbs.*, and $4\frac{3}{16}$ *lbs.*

7. Add $3\frac{1}{4}$ *yds.*, $7\frac{1}{8}$ *yds.*, $5\frac{3}{16}$ *yds.*, and $4\frac{1}{2}$ *yds.*

8. How many *yds.* in 4 pieces of cloth measuring respectively $27\frac{1}{2}$ *yds.*, $37\frac{3}{4}$ *yds.*, $39\frac{1}{5}$ *yds.*, and $30\frac{9}{10}$ *yds.*?

9. How many tons of coal in 5 loads weighing respectively $1\frac{1}{8}$, $1\frac{2}{5}$, $1\frac{3}{10}$, $\frac{1}{2}$, and $1\frac{1}{4}$ tons?

10. How many dollars will pay for a coat worth $\$14\frac{3}{4}$, a hat worth $\$5\frac{1}{2}$, a vest worth $\$6\frac{1}{4}$, a pair of pants worth \$8, and a pair of boots worth $\$9\frac{1}{4}$?

11. How many *pounds* of butter in 4 tubs, weighing respectively $27\frac{1}{3}$ *lbs.*, $34\frac{3}{8}$ *lbs.*, $32\frac{4}{5}$ *lbs.*, and $29\frac{2}{9}$ *lbs.*?

What is addition? What kinds of fractions can be added together? Give the rule for addition of fractions.

SUBTRACTION OF FRACTIONS.

130. Subtraction of Fractions is the operation of finding the difference between two fractions.

James has $\frac{2}{3}$ of an orange; if he gives $\frac{1}{3}$ to his sister, what part of the orange has he left?

ILLUSTRATION. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$.

Charles has $\frac{3}{4}$ of a dollar and gives a poor woman $\frac{1}{4}$ of a dollar; what part of a dollar has he left?

ILLUSTRATION. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

Henry bought $\frac{2}{3}$ of a pound of raisins and gave William $\frac{1}{2}$ of a pound; what part of a pound had he left?

Reduce $\frac{2}{3}$ and $\frac{1}{2}$ to sixths. $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$.

What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Reduce $\frac{1}{3}$ to sixths. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.

From the above illustrations we have the following

RULE.

I. When the fractions have the same denominator, subtract the less numerator from the greater, and place the difference over the common denominator.

II. When they have not the same denominator, reduce them to a common denominator, and subtract as before.

Examples for Oral Work.

1. From $\frac{3}{4}$ subtract $\frac{1}{2}$.
2. From $\frac{5}{6}$ take $\frac{2}{5}$.
3. From $\frac{7}{11}$ take $\frac{2}{7}$.
4. From $\frac{17}{20}$ take $\frac{4}{5}$.
5. From $\frac{3}{5}$ take $\frac{1}{3}$.
6. From $\frac{1}{2}$ take $\frac{1}{5}$.
7. From $\frac{2}{9}$ take $\frac{1}{7}$.
8. From $\frac{23}{24}$ take $\frac{43}{48}$.
9. From $\frac{17}{2}$ take $\frac{4}{9}$.
10. From $\frac{22}{5}$ take $\frac{7}{3}$.
11. From $\frac{19}{9}$ take $\frac{11}{10}$.
12. From $\frac{17}{30}$ take $\frac{2}{5}$.

When there are mixed numbers, change both to improper fractions, and subtract as before.

Examples for Written Work.

13. From $\frac{2}{7}$ take $\frac{4}{15}$.
14. From $3\frac{7}{15}$ take $1\frac{2}{9}$.
15. From $5\frac{7}{25}$ take $2\frac{7}{35}$.
16. From $2\frac{5}{6}$ take $1\frac{7}{12}$.
17. From 7 take $\frac{2}{5}$.
18. From $3\frac{3}{7}$ take $1\frac{4}{5}$.

PRACTICAL PROBLEMS.

1. Find the difference between $\$\frac{7}{8}$ and $\$\frac{1}{4}$. *Ans.* $\$\frac{5}{8}$.

2. I had $\$2\frac{1}{4}$, and spent $\$1\frac{1}{8}$; how much had I left?

3. A farmer had $34\frac{1}{2}$ *tons* of hay, and sold $11\frac{7}{8}$ *tons*; how much had he left?

4. What is the difference between $8\frac{1}{3}$ *yds.* and $3\frac{5}{6}$ *yds.*?

5. The sum of two numbers is $12\frac{7}{16}$; one of the numbers is $7\frac{4}{7}$; what is the other?

6. James, Joseph, and Daniel owned $475\frac{9}{10}$ acres. Daniel had 16 acres more than $\frac{1}{2}$ of it, Joseph 24 acres more than $\frac{1}{3}$ of it, and James the remainder. How many acres had each?

7. A tailor had a piece of cloth containing $29\frac{1}{2}$ *yds.*; he cut off $4\frac{1}{2}$ *yds.* to make a coat and $1\frac{1}{2}$ *yds.* to make a pair of pants. How many *yds.* were left?

8. A housekeeper bought 6 mahogany chairs for $\$23\frac{1}{8}$, and gave for them, 2 ten-dollar and 1 five-dollar bill; what change ought she to receive?

What is subtraction? What fractions can be subtracted? Give the rule for subtraction of fractions. When there are mixed numbers, what is the method?

MULTIPLICATION OF FRACTIONS.

131. Multiplication of Fractions is the operation of finding the product of two factors when one or both are fractions.

132. *To multiply a fraction by a whole number.*

What part of an apple is twice $\frac{1}{4}$ of it?

Solution. $\frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$. (Art. 119, Principle 1.)

What part of an orange is 3 times $\frac{1}{4}$ of it?

Solution. $\frac{1}{4} \times 3 = \frac{3}{4}$.

If each of 4 boys gives me $\frac{1}{8}$ of a pear, what part of a pear shall I then have?

Solution. $\frac{1}{8} \times 4 = \frac{4}{8} = \frac{1}{2}$.

If we had divided the denominator by 4, we should have obtained the same final result. Art. 119, Principle 4.

RULE.

Multiply the numerator by the whole number and write the product over the denominator, or divide the denominator by the whole number and write the quotient under the numerator.

EXAMPLES FOR ORAL WORK.

1. If one yard of cloth cost $\frac{5}{8}$ of a dollar, what will 4 yards cost?

2. Multiply $\frac{47}{49}$ by 7. *4.* Multiply $\frac{127}{15}$ by 5.

3. Multiply $\frac{175}{45}$ by 9. *5.* Multiply $\frac{369}{147}$ by 3.

EXAMPLES FOR WRITTEN WORK.

6. If 1 dollar will buy $\frac{5}{9}$ of a cord of wood, how much will 15 dollars buy? How much 18 dollars?

7. At $\frac{5}{6}$ of a dollar a pound, what will 12 pounds of tea cost? What will 24 *lbs.* cost?

8. If a horse eats $\frac{3}{4}$ of a bushel of oats in a day, how much will 18 horses eat in a day?

9. What will 64 pounds of cheese cost, at $\frac{3}{22}$ of a dollar a pound? What will 66 *lbs.* cost?

NOTE.—When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results; or, reduce the mixed number to an improper fraction, and multiply.

10. At $2\frac{3}{5}$ cents a pound, what will 8 pounds of chalk cost? What will 10 *lbs.* cost?

11. If a man earns $3\frac{9}{10}$ dollars a day, how much will he earn in 15 days? How much in 21 days?

12. If a family consumes $5\frac{4}{7}$ barrels of flour in 1 year, how much would they consume in 9 years? In 12 years?

133. ***To multiply a whole number by a fraction.***

At $2 a yard, what will $\frac{1}{2}$ yard of cloth cost?

SOLUTION. $\frac{1}{2}$ of $2 = 2 \times \frac{1}{2} = \frac{2}{2} = 1$.

At $15 a ton, what will $\frac{4}{5}$ of a ton of hay cost?

SOLUTION.

$$15 \times \frac{4}{5} = \frac{15}{5} \times 4 = 12.$$

EXPLANATION.—1st. Four *fifths* of a ton will cost 4 times as much as 1 *fifth* of a ton; if 1 ton cost 15 dollars, 1 *fifth* will cost $\frac{1}{5}$ of 15 dollars, or 3 dollars, and $\frac{4}{5}$ will cost 4 times 3 dollars, which are 12 dollars. Hence the

RULE.

Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator;

Or, *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

NOTE.—Cancel, when possible.

EXAMPLES FOR MENTAL WORK.

1. What is the product of $9 \times \frac{1}{3}$? $8 \times \frac{1}{2}$? $12 \times \frac{1}{4}$?

2. What is the product of $20 \times \frac{1}{4}$? $28 \times \frac{1}{4}$? $25 \times \frac{1}{5}$?

3. What is the product of $63 \times \frac{3}{7}$? Of $45 \times \frac{4}{9}$?

4. What is the product of $28 \times \frac{3}{4}$? Of $25 \times \frac{2}{5}$?

EXAMPLES FOR WRITTEN WORK.

5. Multiply 33 by $\frac{5}{8}$. *9.* Multiply 65 by $\frac{6}{7}$.

6. Multiply 37 by $\frac{5}{6}$. *10.* Multiply 58 by $\frac{7}{8}$.

7. Multiply 191 by $\frac{2}{9}$. *11.* Multiply 466 by $\frac{6}{7}$.

8. Multiply 525 by $\frac{3}{5}$. *12.* Multiply 646 by $\frac{7}{8}$.

To multiply a whole number by a mixed number, multiply by the fraction and then by the integral part, and find the sum of the products.

13. Multiply 981 by $4\frac{1}{3}$.

ILLUSTRATION.

$$\begin{array}{r} 3)981 \\ 4\frac{1}{3} \\ \hline 327 \\ 3924 \\ \hline 4251 \end{array}$$

EXPLANATION.—Multiplying 981 by $\frac{1}{3}$ is the same as dividing it by 3; this gives 327; multiplying 981 by 4, we find 3924, and the sum of the results is equal to 4251, which is the required product.

14. Multiply 67 by $9\frac{1}{2}$.

15. Multiply 9 by $12\frac{2}{3}$.

16. Multiply 108 by $12\frac{4}{9}$.

17. Multiply 1,464 by $3\frac{1}{4}$.

134. ***To multiply one fraction by another.***

1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

ILLUSTRATION.

$$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}.$$

EXPLANATION.—To multiply $\frac{3}{4}$ by $\frac{5}{7}$, we multiply $\frac{3}{4}$ by 5, and then divide the result by 7. To multiply $\frac{3}{4}$ by 5, we multiply its numerator by 5, (Art. 119, Prin. 1); to divide the result by 7, we multiply the denominator by 7, (Art. 119, Prin. 3), that is, we multiply 3 by 5 and 4 by 7.

From this illustration and explanation we deduce the following

RULE.

Reduce the fractions (when necessary) to equivalent simple ones; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

EXAMPLES FOR ORAL WORK.

2. Multiply $\frac{2}{5}$ by $\frac{4}{7}$. *Ans.* $\frac{8}{35}$.

3. Multiply $\frac{5}{3}$ by $\frac{2}{7}$.

4. Multiply $\frac{3}{8}$ by $\frac{5}{7}$.

5. Multiply $\frac{3}{2}$ by $\frac{5}{7}$.

6. Multiply $\frac{3}{5}$ by $\frac{7}{5}$.

7. Multiply $\frac{3}{5}$ by $\frac{3}{5}$.

8. Multiply $\frac{5}{2}$ by $\frac{7}{3}$.

9. Multiply $\frac{12}{7}$ by $\frac{3}{8}$.

10. Multiply $\frac{7}{36}$ by $\frac{24}{17}$.

11. Multiply $\frac{100}{27}$ by $\frac{15}{25}$.

12. Multiply $\frac{28}{3}$ by $\frac{36}{7}$.

EXAMPLES FOR WRITTEN WORK.

If there are any factors common to the numerator and the denominator of the indicated product, cancel them before performing the multiplication.

13. What is the product of $\frac{1}{2} \times \frac{7}{8} \times \frac{4}{5} \times \frac{9}{4}$?

14. Find the result of $\frac{9}{10} \times \frac{2}{3} \times 1\frac{1}{4}$.

15. What is the product of $\frac{3}{8} \times \frac{6}{7} \times \frac{7}{9} \times \frac{9}{11} \times \frac{11}{12} \times \frac{15}{17}$?

16. Find the product of $\frac{5}{9} \times \frac{13}{15} \times \frac{3}{5} \times \frac{9}{20} \times \frac{6}{7}$.

RULE FOR REDUCING A COMPOUND FRACTION.

A compound fraction is reduced to a simple one by multiplying its numerators together for a numerator, and the denominators together for a denominator.

17. Reduce to a simple fraction $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$.

18. Reduce $\frac{1}{7}$ of 14 of $\frac{3}{5}$ of 15 of $\frac{6}{7}$ to a simple fraction.

Multiply

19. $\frac{1}{8}$ of 3, by $\frac{1}{6}$ of $15\frac{1}{9}$.

20. $\frac{2}{3}$ of $\frac{5}{2}$ of $\frac{3}{5}$ by $4\frac{1}{6}$.

21. $14\frac{5}{6}$ of 9, by $6\frac{3}{7}$.

22. $\frac{3}{7}$ of 6 of $\frac{5}{4}$, by $\frac{8}{5}$ of 4.

23. $\frac{1}{9}$ of $\frac{3}{5}$ of $\frac{7}{8}$ of 40.

24. $\frac{1}{2}$ of $5\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{1}{21}$.

25. $\frac{9}{11}$ of $\frac{44}{3}$ of $\frac{7}{20}$ of 18.

26. $\frac{6}{13}$ of $4\frac{1}{2}$ of $\frac{3}{23}$ of 9.

To multiply two mixed numbers together, reduce both to simple fractions, and then multiply by the rule for multiplication of fractions.

27. Multiply $7\frac{1}{2}$ by $7\frac{1}{2}$. *Ans.* $\frac{15}{2} \times \frac{15}{2} = 56\frac{1}{4}$.

28. Multiply $2\frac{2}{3}$ by $4\frac{1}{2}$. *Ans.* $\frac{8}{3} \times \frac{9}{2} = 12$.

29. Multiply $4\frac{1}{8}$ by $3\frac{7}{11}$.

30. Multiply $3\frac{2}{3}$ by $5\frac{1}{9}$.

31. Multiply $12\frac{1}{2}$ by $87\frac{1}{5}$.

32. Multiply 643 by $4\frac{2}{3}$.

What is multiplication of fractions? Give the rule for multiplying a fraction by a whole number. When the multiplicand is a mixed number, how do you proceed? Give the rule for multiplying a whole number by a fraction. A whole number by a mixed number. One fraction by another. If there are factors common to both terms, what should be done? How is a compound fraction reduced to a simple one? How multiply two mixed numbers together?

PRACTICAL PROBLEMS.

1. What will $2\frac{1}{2}$ bushels of corn cost at $\$1\frac{1}{2}$ per bushel?

2. A man owned $\frac{2}{3}$ of a farm and sold $\frac{5}{8}$ of his interest; what part of the farm did he sell? *Ans.* $\frac{5}{12}$.

3. What is the cost of $37\frac{1}{2}$ *lbs.* of lard at $17\frac{1}{2}$ *cts.* a *lb.*?

4. Find the value of 876 bushels of oats at $62\frac{1}{2}$ *cts.* a *bu.*

5. What is the cost of $16\frac{3}{4}$ *yds.* of ribbon at $62\frac{1}{2}$ *cts.* a *yd.*?

6. What is the cost of $19\frac{1}{2}$ *yds.* of muslin at $22\frac{1}{2}$ *cts.* a *yd.*?

7. A farmer sold $151\frac{1}{4}$ bushels of wheat at $\$1\frac{1}{2}$ per bushel; how much did he receive for it?

8. A grocer sold $36\frac{1}{4}$ *lbs.* of tea at $87\frac{1}{2}$ *cts.* a pound; how much did it bring?

9. If a man can earn $\$2\frac{1}{2}$ per day, how much can he earn in $31\frac{1}{4}$ days?

10. What is the product of $6\frac{1}{5}$, $2\frac{7}{9}$, and $\frac{1}{4}$ of 12?

11. What will 24 yards of cloth cost, at $\$3\frac{3}{4}$ a yard?

12. What will $6\frac{2}{3}$ bushels of wheat cost, at $\$3\frac{3}{4}$ a bushel?

13. A horse eats $\frac{3}{14}$ of $\frac{7}{9}$ of 12 tons of hay in three months; how many tons does he consume?

14. If $\frac{2}{3}$ of $\frac{5}{8}$ of a dollar buy a bushel of corn, what will $\frac{7}{10}$ of 6 bushels cost?

15. What cost $5\frac{2}{3}$ gallons of molasses, at $96\frac{1}{2}$ *cts.* a gallon?

16. What will $7\frac{11}{12}$ dozen candles cost, at $\$\frac{3}{16}$ per dozen?

DIVISION OF FRACTIONS.

135. Division of Fractions is the operation of finding the quotient when either the divisor or dividend, or both, are fractions.

136. *To divide a fraction by a whole number.*

1. If 2 bushels of potatoes cost $\$\frac{4}{5}$, what will 1 bushel cost?

1 bushel will cost $\frac{1}{2}$ as much as 2 bushels, $= \frac{2}{5}$ of a dollar.

2. If 3 bushels of apples cost $\frac{6}{7}$ of a dollar, what will 1 bushel cost? $\frac{6}{7} \div 3 = \frac{2}{7}$. (Art. 119, Prin. 2.)

3. If 4 quarts of beans cost $\frac{2}{3}$ of a dollar, what will 1 quart cost? $\frac{2}{3} \div 4 = \frac{2}{12} = \frac{1}{6}$. (Art. 119, Prin. 3.)

RULE,

Divide the numerator by the whole number, and write the quotient over the denominator; or, multiply the denominator by the whole number, and write the product under the numerator.

EXAMPLES FOR ORAL WORK.

1. Divide $\frac{12}{11}$ by 6.

2. Divide $\frac{18}{37}$ by 9.

3. Divide $\frac{18}{19}$ by 6.

4. Divide $\frac{24}{91}$ by 12.

5. If 6 horses eat $\frac{9}{10}$ of a ton of hay in 1 month, how much does each horse eat?

6. If 9 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost? What will 3 *yds.* cost?

7. If 1 yard of cloth cost 4 dollars, how much can be bought for $\frac{8}{9}$ of a dollar? How much for $\frac{11}{9}$ of a dollar?

8. If 5 pounds of coffee cost $\frac{15}{16}$ of a dollar, what will 1 pound cost? What will 2 *lbs.* cost?

9. At \$6 a barrel, what part of a barrel of flour can be bought for $\frac{3}{5}$ of a dollar? For \$$\frac{4}{5}$? For \$11?

10. If 10 bushels of barley cost $3\frac{1}{3}$ dollars, what will 1 bushel cost? What will 4 bushels cost?

EXAMPLES FOR WRITTEN WORK.

1. Divide $\frac{405}{19}$ by 15.
2. Divide $\frac{450}{531}$ by 75.
3. Divide $\frac{15}{33}$ by 20.
4. Divide $\frac{126}{651}$ by 27.
5. Divide $\frac{561}{784}$ by 25.
6. Divide $\frac{172}{395}$ by 13.
7. Divide $3\frac{1}{8}$ by 15.
8. Divide $4\frac{1}{2}$ by 16.

9. If 21 pounds of raisins cost $4\frac{2}{3}$ dollars, what will 1 pound cost? What will $4\frac{1}{2}$ *lbs.* cost? $7\frac{3}{4}$ *lbs.*?

10. If 12 men consume $6\frac{2}{5}$ pounds of meat in a day, how much does 1 man consume?

NOTE.—Reduce the mixed numbers to improper fractions, and divide as in the case of a simple fraction.

137. ***To divide a whole number by a fraction.***

1. How many halves in 1 apple?

2. How many fourths in 1 apple?

3. How many fifths in 1 apple?

4. How many $\frac{1}{2}$ bushels of wheat can we empty into a box which will hold 2 bushels?

Since we can empty 1 bushel 2 times, we can empty $\frac{1}{2}$ bushel twice 2 times, or 4 times. $2 \div \frac{1}{2} = 2 \times \frac{2}{1} = 4$.

RULE.

I. Invert the terms of the divisor, and multiply the whole number by the resulting fraction; or,

II. Divide the dividend by the numerator of the divisor, and multiply the quotient by the denominator.

EXAMPLES FOR ORAL WORK.

1. Divide 6 by $\frac{2}{3}$. $6 \div \frac{2}{3} = \frac{6}{2} \times 3 = 9$.
2. Divide 12 by $\frac{6}{7}$.
3. Divide 18 by $\frac{9}{11}$.
4. Divide 25 by $\frac{5}{7}$.
5. Divide 36 by $\frac{12}{13}$.

6. At $\frac{11}{12}$ of a dollar a yard, how many yards of cloth can be bought for 9 dollars?

7. If a man travel $\frac{7}{8}$ of a mile in one hour, how long will it take him to travel 10 miles?

8. If $\frac{5}{8}$ of a ton of hay is worth 9 dollars, what is a ton worth?

EXAMPLES FOR WRITTEN WORK.

1. Divide 17 by $\frac{11}{5}$.
2. Divide 100 by $\frac{10}{13}$.
3. Divide 500 by $\frac{3}{8}$.
4. Divide 27 by $\frac{9}{10}$.
5. Divide 750 by $\frac{100}{147}$.
6. Divide 100 by $6\frac{7}{10}$.
7. Divide 75 by $10\frac{2}{3}$.
8. Divide 97 by $8\frac{1}{3}$.

138. ***To divide one fraction by another.***

ILLUSTRATION.

$$\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}.$$

EXPLANATION.—Here we have to divide $\frac{3}{7}$ by $\frac{4}{5}$; we multiply $\frac{3}{7}$ by 5 and then divide the result by 4; but this is the same thing as multiplying $\frac{3}{7}$ by $\frac{5}{4}$. Hence, we invert the divisor, that is, we cause its terms to change places, and then multiply the dividend by the result.

RULE.

Reduce mixed numbers and complex fractions to simple ones. Invert the terms of the divisor and proceed as in multiplication.

EXAMPLES FOR ORAL WORK.

1. Divide $\frac{2}{3}$ by $\frac{5}{7}$.

2. Divide $\frac{3}{8}$ by $\frac{9}{11}$.

3. Divide $\frac{2}{9}$ by $\frac{3}{5}$.

4. Divide $\frac{4}{7}$ by $\frac{3}{5}$.

5. How much cheese can be bought for $\frac{7}{8}$ of a dollar, at $\frac{2}{11}$ of a dollar a pound?

EXAMPLES FOR WRITTEN WORK.

1. Divide $\frac{3}{5}$ by $\frac{8}{77}$.

2. Divide $\frac{6}{35}$ by $\frac{4}{9}$.

3. Divide $\frac{1}{2}$ of $\frac{3}{5}$ by $\frac{8}{15}$.

4. Divide $2\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{2}{7}$.

5. Divide $\frac{1}{2}$ of $\frac{3}{5}$ by $\frac{7}{15}$.

6. Divide $2\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{3}{7}$.

7. Divide $\frac{15}{8} \times \frac{36}{5}$ by $\frac{22}{9}$.

8. Divide $7\frac{1}{2}$ by $10\frac{2}{3}$.

9. Divide $9\frac{7}{8}$ by $8\frac{1}{3}$.

10. Divide $\frac{2}{3}$ of 16 by $\frac{1}{2}$ of $\frac{1}{3}$.

COMPLEX FRACTIONS.

139. Complex fractions are reduced to their simplest forms by the operations of Reduction and Division.

1. Reduce $\frac{2\frac{2}{5}}{4\frac{1}{2}}$ to its simplest form.

ILLUSTRATION.

$$\frac{2\frac{2}{5}}{4\frac{1}{2}} \times \frac{10}{10} = \frac{24}{45} = \frac{8}{15}.$$

EXPLANATION. — Multiplying both terms of the complex fraction by 10, the least common multiple of the denominators of the fractions $\frac{2}{5}$ and $\frac{1}{2}$, we have $\frac{24}{45}$, (Art. 119, Prin. 5.) This reduced to its lowest terms is $\frac{8}{15}$. Hence, for the reduction of a complex fraction to its simplest form, we have the following

RULE.

I. Reduce the numerator and denominator each (when necessary) to equivalent simple fractions; then, multiply both terms of the complex fraction by the least common multiple of the denominators: or,

II. Divide the numerator of the complex fraction by the denominator.

EXAMPLES.

1. Reduce $\frac{6\frac{2}{3}}{\frac{2}{3}}$ to a simple fraction. $\frac{6\frac{2}{3} \times 3}{\frac{2}{3} \times 3} = \frac{20}{2} = 10.$

2. Reduce $\frac{7\frac{11}{12}}{3\frac{5}{6}}$ to a simple fraction. $\frac{7\frac{11}{12} \times 12}{3\frac{5}{4} \times 12} = \frac{95}{51}.$

3. Reduce $\frac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}}$ to a simple fraction.

NOTE.—In the third example we reduce the terms of the complex fraction to simple fractions.

SOLUTION. $\frac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}} = \frac{\frac{6}{45} \times 180}{\frac{22}{36} \times 180} = \frac{24}{140} = \frac{12}{55}.$

Reduce the following complex fractions to simple ones.

4. $\frac{2\frac{1}{4}}{3\frac{1}{5}}$.

5. $\frac{3\frac{1}{2}}{4}$.

6. $\frac{7}{4\frac{1}{4}}$.

7. $\frac{8\frac{1}{12}}{3}$.

8. $\frac{14\frac{9}{10}}{\frac{7}{9} \text{ of } 15}$.

9. $\frac{214\frac{3}{4}}{25\frac{11}{12}}$.

10. $\frac{\frac{4}{7} \text{ of } \frac{2}{3} \text{ of } 5\frac{1}{4}}{1\frac{5}{16} \text{ of } 48}$.

11. $\frac{\frac{5}{6} \text{ of } \frac{7}{8} \text{ of } 4\frac{1}{3}}{\frac{8}{9} \text{ of } 27}$.

TEST QUESTIONS.

What is division of fractions? Give the rule for dividing a fraction by a whole number. How do you divide a whole number by a fraction? What is understood by *inverting* the terms of a fraction? Give the rule for dividing one fraction by another. What is a complex fraction? By what operations are complex fractions reduced to simple ones? Give the rule. What is a problem?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

1. How many pairs of gloves can I buy for \$2, at $\frac{1}{3}$ of a dollar a pair? How many for \$3? For \$6?

2. If a man walks 1 mile in $\frac{2}{5}$ of an hour, how many miles will he walk in 1 hour? How many in 2 hours?

3. If a man can hoe $\frac{1}{3}$ of a field of corn in one day, in how many days can he hoe $\frac{3}{4}$ of a field? In how many days can he hoe the whole field?

4. If 5 *yds.* of muslin cost \$$1\frac{1}{8}$, what will 1 *yd.* cost? What will 2 *yds.* cost? What will 10 *yds.* cost?

5. If 5 pounds of coffee cost $\frac{10}{13}$ of a dollar, what will 1 pound cost? What will 2 *lbs.* cost? What 5 *lbs.*?

FOR WRITTEN WORK.

1. How many barrels of apples can be bought for \$$20\frac{1}{2}$, if one barrel costs \$$2\frac{3}{4}$? *Ans.* $7\frac{5}{11}$ *barrels.*

2. How many bags of flour will \$$70\frac{3}{8}$ buy, if each bag costs \$$2\frac{1}{4}$? How many if each bag costs \$$4\frac{1}{2}$?

3. If \$$37\frac{3}{8}$ is divided equally among 6 persons, how much will each receive? \$$37\frac{3}{8} \div 6 = ?$

4. A merchant sold from a piece of cloth $4\frac{1}{2}$ *yds.* to one customer, $5\frac{7}{8}$ to a second, and $3\frac{1}{4}$ to a third; how much did he sell in all?

5. A grocer sold $10\frac{1}{2}$ *lbs.* of sugar at $12\frac{1}{2}$ cents per *lb.*; and $14\frac{1}{4}$ *lbs.* at $16\frac{1}{8}$ cents; what did he receive for it all?

6. A grocer had a barrel of sugar that weighed $348\frac{1}{4}$ *lbs.*, from which he sold $58\frac{1}{4}$ *lbs.* to one customer, $27\frac{1}{2}$ *lbs.* to another, and $64\frac{3}{4}$ *lbs.* to a third; how much had he then remaining?

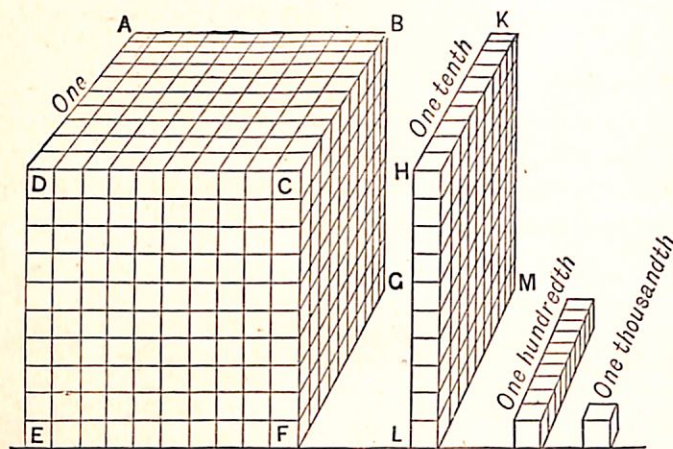
7. What is the cost of $7\frac{1}{2}$ tons of coal, at \$$7\frac{1}{2}$ per ton?

8. If 11 men consume a barrel of flour in $14\frac{1}{2}$ days, how long will $5\frac{1}{4}$ barrels last them?

9. If one man earns \$$2\frac{1}{2}$ in 1 day, and another earns \$$3\frac{1}{4}$ in 1 day, how much will both earn in $3\frac{1}{2}$ days?

10. A dealer buys corn at the rate of \$$\frac{7}{8}$ per bushel, and sells it at \$$1\frac{1}{20}$ per bushel; how much does he make on $142\frac{1}{2}$ bushels?

REVIEW QUESTIONS.

What is a fractional unit? Illustrate. What is a fraction? The denominator? The numerator? Terms? How are common fractions written? How read? Write a fraction in which the integral unit is divided into 7 parts, and 3 of those parts are expressed. Tell which is its numerator and which its denominator. What is a proper fraction? An improper fraction? A mixed number? A simple fraction? A complex fraction? A compound fraction? Write an improper fraction. A mixed number. A proper fraction. Are the proper and improper fractions that you have written simple fractions? Write a complex fraction. Reduce it to a simple fraction. Write a compound fraction. Reduce it to a simple fraction. Is it a proper or an improper fraction? State the six principles used in fractions. What is reduction? Give the rule for reducing a whole number to a simple fraction having a given denominator. A mixed number to a simple fraction. An improper fraction to a mixed number. A simple fraction to its lowest terms. A compound fraction to a simple one. Fractions to their least common denominator. A complex fraction to a simple one. Define addition of fractions. Give the rule for addition of fractions. Define subtraction of fractions. Give the rule for subtraction of fractions. If a boy spends $\frac{1}{3}$ of his money for a book, and $\frac{2}{5}$ of it for a slate, what part of his money has he left? Define multiplication of fractions. Give the rule for multiplying a fraction by a whole number. For multiplying a whole number by a fraction. One fraction by another. Define division of fractions. Give the rule for dividing a fraction by a whole number. A whole number by a fraction. A fraction by a fraction.

FORMATION OF DECIMAL FRACTIONS.

140. If a block is divided into 10 equal parts, each part will be $\frac{1}{10}$ of the block.

If one of the equal parts is divided into 10 equal parts, each of the parts will be $\frac{1}{100}$ of the original block; $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

If one of the last parts is again divided into 10 equal parts, each part will be $\frac{1}{1000}$ of the original block; $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{1000}$.

EXAMPLES.

1. If an apple is divided into 10 equal parts, what is each part called? $\frac{1}{10}$ of $1 = ?$

2. If $\frac{1}{10}$ of an apple is divided into 10 equal parts, what is each part called? $\frac{1}{10}$ of $\frac{1}{10} = ?$

Write each of the parts you have mentioned.

3. What part of one is $\frac{1}{10}$ of $\frac{1}{100}$? Write it.

4. What part of one is $\frac{1}{10}$ of $\frac{1}{1000}$? Write it.

These fractions are called decimal fractions.

DEFINITIONS.

141. A **decimal fraction** is a fraction whose denominator is 1, with one or more ciphers annexed.

One of the decimal divisions of an integral unit is a **decimal unit,** as $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

142. ***Manner of writing Decimals.***

Integers increase from the right to the left, and decrease from the left to the right by a *scale of tens.*

In the integral number 1111, 1 hundred is $\frac{1}{10}$ of 1 thousand, 1 ten is $\frac{1}{10}$ of 1 hundred, and 1 unit is $\frac{1}{10}$ of 1 ten.

We will place a point (.) after the unit figure, and continue the scale; the next expression is $\frac{1}{10}$, which we indicate by 1 on the right of the point, thus 1111.1, and read it one-tenth; the next is $\frac{1}{100}$, which we indicate by 1 in the second place on the right of the point, thus 1111.11, and read *it* one-hundredth; $\frac{1}{1000}$ is expressed by 1 in the third place on the right of the point, and so on.

We have two methods of writing decimal fractions.

1. Both numerator and denominator may be written.

2. The numerator alone may be written, and the denominator may be indicated by a point, (.), called the **decimal point.**

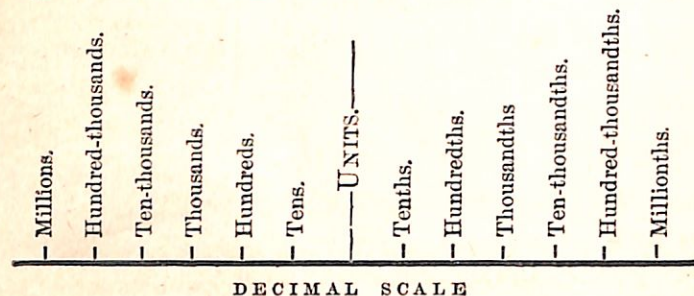
NOTATION OF DECIMALS.

143. To write a decimal fraction by the second method, we write the numerator; and if the number of figures is equal to the number of ciphers in the denominator, prefix the decimal point, Thus $\frac{3}{10} = .3$, $\frac{32}{100} = .32$. If the number of figures is less than the number of ciphers in the denominator, we make them equal by prefixing ciphers, and then prefix the decimal point.

Thus, $\frac{3}{100} = .03$, $\frac{52}{1000} = .052$.

Ciphers that immediately follow the decimal point are called **prefixed ciphers.** Ciphers that follow the last significant figure are called **terminal ciphers.**

An improper decimal fraction, written by means of the decimal point, is expressed as a mixed number; $\frac{23}{10}$ is written thus, 2.3, and read two and three-tenths. Such expressions are called **Mixed Decimals.**

DECIMAL SCALE

RULE FOR NOTATION OF DECIMALS.

Write the number of tenths in the first decimal place, the number of hundredths in the second place, the number of thousandths in the third place, and so on.

EXAMPLES

Write the following fractions, omitting the denominators.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$\frac{4}{10}$.	$\frac{3}{100}$.	$\frac{12}{1000}$.	$\frac{113}{10000}$.	$\frac{2}{10000}$.	$\frac{372}{10}$	$\frac{1345}{100}$.

144. ***Numeration of Decimals.***

ILLUSTRATION.

Tenths.	Hundredths.	Thousandths.	10-thousandths.	100-thousandths.	Millionths.
.0	0	0	3	7	5

To read .000375, we commence at the decimal point and numerate, as shown in the illustration. The lowest order of units indicates the denominator; we read it 375 *millionths.*

Every fraction has a numerator and a denominator expressed or implied.

When a decimal fraction is written by means of the decimal point, its denominator is indicated by the lowest decimal unit; hence, to read decimals requires two numerations, one from the left to the right to determine the denominator, and one from the right to the left to read the numerator.

RULE FOR READING DECIMALS.

Read the numerator as if it were an integer, naming its denominator from the decimal unit of the right-hand figure.

EXAMPLES.

Read the following decimals.

1. .003.	*4.* .005.	*7.* .50103.	*10.* .001,325.
2. .07.	*5.* .032.	*8.* 20256.	*11.* 7.814,3.
3. .000123.	*6.* .314.	*9.* .135.	*12.* 6.010406.

Write the following numbers in figures, then numerate them, and read them.

13. Forty-one, and *three / tenths.*

14. Sixteen, and *three / millionths.*

15. Five, and *nine / hundredths.*

16. Sixty-five, and *fifteen / thousandths.*

17. Eighty, and *three / millionths.*

18. Two, and *three / hundred millionths.*

19. Four hundred, and *ninety-two / thousandths.*

20. Thirty-nine, and *six hundred and forty / thousandths.*

21. Three thousand, and *eight hundred and forty / millionths.*

22. *Six hundred and fifty / thousandths.*

NOTE.—In the preceding examples, decimals are in *italics*. The sign / separates the numerator from the denominator.

PRINCIPLES.

145. From the nature of decimals we also have the following principles:

1. *Annexing ciphers to a decimal does not change its value.*

2. *Removing ciphers from the right of a decimal does not change its value.*

3. *Annexing decimal ciphers to an integer does not change its value.*

What is a decimal fraction? One hundred is what part of one thousand? Ten is what part of one hundred? One is what part of ten? One-*tenth* is what part of one? One hundredth is what part of one-tenth? One-thousandth is what part of one-hundredth? Write a decimal scale that will show the order of units from one-millionth to one million inclusive. How many methods have we of writing decimal fractions? Illustrate and explain both methods. Give the rule for notation of decimals. Numerate the decimal .234758. Read it. Give the rule for reading decimals. State the three principles given in Art. 145.

ADDITION OF DECIMALS.

146. Addition of decimals is the operation of finding the sum of two or more decimals.

1. What is the sum of $\frac{1}{10}$ and $\frac{3}{10}$? $\frac{1}{10} + \frac{3}{10} = \frac{4}{10}$. This expressed by means of the decimal point, is written thus, .4, and read four-tenths.

2. Add .1, .3, and .5. $.1 + .3 + .5 = .9$.

3. What is the sum of .21 and .35? $.21 + .35 = .56$.

4. If an arithmetic cost 5 tenths of a dollar, and a grammar 8 tenths, what do they both cost?

$$\frac{5}{10} + \frac{8}{10} = \frac{13}{10} = 1.3.$$

5. What is the sum of 37.04, 704, 3 and .0376?

ILLUSTRATION.

```
  37.0400
 704.3000
    .0376
 --------
 741.3776
```

EXPLANATION.—We write the numbers so that units of the same order stand in the same column; this brings the decimal points also in the same column. We commence at the right and add as in integers; bring down the decimal point and place it beneath, at the left of tenths' place.

RULE.

Write the decimals so that units of the same order shall stand in the same column and add as in simple numbers; and place the decimal point in the amount under the decimal points in the numbers added.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
.314	.23	.318	8.314
.2121	.375	.2647	2.002
.034	.4143	.03	3.01

(5.)	(6.)	(7.)	(8.)
3.002	1.814	274.3	35.7
14.07	2.3006	71.07	4.115
17.0143	4.05	80.013	19.
3.2	7.1	40.7112	9.0035

9. Add .7, .07, .175, and 2.0325.

10. Add 17.215, 3.056. .009, and 2.079.

11. Add 351.8, 45.001, 2.7169, and 34.21.

12. Add 3, 29.157, 8.116, and 123.49.

13. Add 25.29, 367.5, 2.091, and 55.

14. Add 21.03, 34.72, 5.005, and 25.1.

15. Add 31.91, 21.375, 4.03, and .005.

16. Add 3.042, 8.7562, 0.1437, and 0.021.

PRACTICAL PROBLEMS.

1. A man bought a horse for $203.75, a buggy for $305.50, a harness for $43.25, and a whip for $3.75; what was the cost of the whole? *Ans.* $556.25.

2. A farm consists of 33.125 acres of woodland, 24.876 acres of pasture land, 14.886 acres of meadow, and 13.888 acres of plough land; how many acres in the farm?
Ans. 86.775 acres.

3. A farmer sold a pair of horses for $287.755, a yoke of oxen for $173.865, and 7 cows for $213.825; what did he receive for them all?

4. A man bought a lot for $874.75, built a house on it that cost $5,843.795, a barn that cost $563.254, and then expended $2,811.345 for furniture; what was the cost of the whole?

What is addition of fractions? Give the rule for addition of decimal fractions.

SUBTRACTION OF DECIMALS.

147. Subtraction of Decimals is the operation of finding the difference between two decimal numbers.

1. A boy having 7 tenths of a dollar, paid 2 tenths for a trip to the park; how much had he left?

$\frac{2}{10}$ from $\frac{7}{10}$ leaves $\frac{5}{10}$, which expressed by means of the decimal point is .5.

2. From $\frac{25}{100}$ subtract $\frac{11}{100}$. $\frac{25}{100} - \frac{11}{100} = \frac{14}{100} = .14$.

3. From .8 subtract .3. $.8 - .3 = .5$.

4. From .78 subtract .5.

$$\frac{5}{10} = \frac{50}{100}; \quad .78 - .50 = .28.$$

5. From 56.403 subtract 18.6.

ILLUSTRATION.

56.403
18.6
37.803

EXPLANATION. — We write the subtrahend under the minuend, so that units of the same order shall stand in the same column; this will bring the decimal points in the same column. We then subtract as in simple numbers. Hence the following

RULE.

Write the subtrahend under the minuend so that units of the same order shall stand in the same column; then subtract as in simple numbers and place the decimal point in the remainder under the decimal points of the subtrahend and minuend.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
4.302	15.005	4.5301	5.6747
2.109	3.114	2.107	0.0328
2.193	11.891	2.4231	5.6419

(5.)	(6.)	(7.)	(8.)
37.290	8.41	102.	210.
18.143	3.987	3.884	.0014
19.147	4.423	98.116	209.9986

9. From 18.47 take 2.031.

10. From 13.81 take 8.492.

11. From 11.1156 subtract 2.0301.

12. From 1.805 subtract 0.0184.

13. $4.443-3.999=?$

14. $8.123-6.015=?$

15. $17.41-14.14=?$

16. $120-25.75=?$

17. 376.403 take 143.709.

18. 1,334.5 take 20.7362.

19. 8.0307 take 0.087.

20. 92.92 take 29.29.

PRACTICAL PROBLEMS.

1. A man had $18.75, but spent $4.755; how much had he left? *Ans.* $13.995.

2. A farmer bought groceries to the amount of $117.743, of which he paid $93.817 in oats, and the remainder in cash; how much did he pay in cash? *Ans.* $23.926.

3. A dealer bought goods for $8,743.85, and sold them for $11,342.81; how much did he gain?

4. Two fields contain 87.3142 acres, and the smaller one contains 32.8954 acres; what does the larger one contain?

5. From a hogshead of sugar weighing 993.142 *lbs.*, 418.387 *lbs.* were sold; how much remained?

TEST QUESTIONS.

What is subtraction of decimals? What kind of fractions can be subtracted? Give the rule for subtraction of decimals. What is the minuend? What the subtrahend? What is the result in subtraction?

MULTIPLICATION OF DECIMALS.

148. ***Multiplication of decimals is the operation of finding the product of two decimals.***

1. Multiply $\frac{1}{10}$ by 3.

$\frac{1}{10} \times 3 = \frac{3}{10}$ expressed by means of the decimal point, .3.

2. Multiply $\frac{2}{10}$ by $\frac{3}{10}$, $\frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = .06$.

3. Multiply $\frac{3}{10}$ by $\frac{3}{100}$, $\frac{3}{10} \times \frac{3}{100} = \frac{9}{1000} = .009$.

4. Multiply $\frac{4}{10}$ by $\frac{2}{1000}$, $\frac{4}{10} \times \frac{2}{1000} = \frac{8}{10000} = .0008$.

If *tenths* are multiplied by *units*, the product is *tenths.*

If *tenths* are multiplied by *tenths*, the product is *hundredths.*

If *tenths* are multiplied by *hundredths*, the product is *thousandths.*

If *tenths* are multiplied by *thousandths*, the product is *ten thousandths.*

The denominator of the product will therefore contain as many ciphers as there are ciphers in the denominators of the factors. If the decimal be expressed by means of the decimal point, the number of decimal figures is still determined by the number of ciphers in the denominator; hence, the number of decimal figures in the product must equal the number of decimal figures in the factors.

5. Multiply .5 by 7.

Since tenths multiplied by units give tenths, $.5 \times 7$ units $= 35$ tenths $= 3.5$.

6. Multiply .3 by .2.

Since tenths multiplied by tenths give hundredths, $.3 \times .2 = .06$.

7. Multiply .5 by .09.

Since tenths multiplied by hundredths give thousandths, $.5 \times .09 = .045$.

PRINCIPLE.—*There will be as many decimal places in the product as there are in the factors.*

RULE.

Multiply as in integers, and point off from the right of the product, as many decimal figures as there are in both factors.

Multiply the following

1. 4.7 by 3.2.	*8.* 3.12 by 2.4.	*15.* 402 by 0.43.
2. 21.2 by 0.12.	*9.* 86.28 by 2.5.	*16.* 99 by .25.
3. 17.4 by 2.3.	*10.* 7.81 by 3.25.	*17.* .845 by 16.
4. 3.18 by .34.	*11.* 1.057 by 1.032.	*18.* 175.6 by 3.24.
5. 3.51 by .75.	*12.* 3.33 by 2.22.	*19.* 38.42 by 3.27.
6. 4.81 by 8.2.	*13.* 8.19 by .034.	*20.* 56.57 by 12.35.
7. .03 by .12.	*14.* 0.143 by .12.	*21.* 387.2 by 8.143.

To multiply a decimal or a mixed decimal by 10, 100, 1000, etc., *move the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand if necessary.*

Multiply

22. .78 by 10.	*24.* .473 by 1000.
23. 8.42 by 100.	*25.* 6.5 by 100.

PRACTICAL PROBLEMS.

1. If a person earns \$3.5 in 1 day, how much will he earn in 4.3 days? *Ans.* \$15.05.

2. What will 36.75 *lbs.* of butter cost at 19.75 cents per *lb.*? *Ans.* \$7.2581.

3. What will 17.3 tons of coal cost, at \$7.75 per ton?

4. If a boat sails 7.35 miles in 1 hour, how far will she sail in 3.12 hours? How far in 5.25 hours?

5. If a barrel of flour weighs 203.5 *lbs.*, what will 7.33 barrels weigh? What will 8.05 barrels weigh?

6. What will 3.8 acres of land cost, at \$55.72 per acre?

7. A merchant sold 7.3 *yds.* of cloth at \$2.32 per yard; what did he receive for it?

8. There are 5.5 yards in 1 rod; how many yards are there in 71.24 rods? In 84.05 rods?

9. What is the cost of 51.3 gallons of wine, at \$3.125 per gallon? What at \$4.25 a gallon?

10. Find the value of 7.1 *lbs.* of meat, at 16.21 cents a pound. Find the value of 15.5 *lbs.*

11. If a man travels at the rate of 18.7 miles per day, how far will he travel in 7.11 days?

12. What will 8.5 *lbs.* of beef cost, at 22.5 cents a pound? What will 20.25 *lbs.* cost?

TEST QUESTIONS.

If tenths are multiplied by units, what denomination is the product? If tenths are multiplied by tenths, what is the product? If tenths are multiplied by hundredths, what is the product? If tenths are multiplied by thousandths, what is the product? Give the rule for multiplication of decimals. How do you multiply by 10, 100, 1,000, etc.

DIVISION OF DECIMALS.

149. Division of Decimals is the operation of finding the quotient of one decimal by another.

1. Divide 5 apples equally among 5 boys.

$5 \div 5 = 1$ (the fifth of 5).

2. Divide .5 of an apple equally among 5 boys.

$.5 \div 5 = .1$ (the fifth of .5).

3. Divide .05 of a dollar equally among 5 boys.

$.05 \div 5 = .01$ (the fifth of .05).

4. Divide .005 by 5.

$.005 \div 5 = .001$ (the fifth of .005).

5. Divide .0005 by 5.

$.0005 \div 5 = .0001$ (the fifth of .0005).

6. Divide .5 by .5.

Reduce the decimal to the form of a common fraction.

$.5 \div .5 = \frac{5}{10} \div \frac{5}{10} = \frac{5}{10} \times \frac{10}{5} = 1.$

7. Divide .05 by .5.

$.05 \div .5 = \frac{5}{100} \div \frac{5}{10} = \frac{5}{100} \times \frac{10}{5} = \frac{1}{10} = .1.$

8. Divide .005 by .5.

$.005 \div .5 = \frac{5}{1000} \div \frac{5}{10} = \frac{5}{1000} \times \frac{10}{5} = .01.$

9. Divide .0005 by .5.

$.0005 \div .5 = \frac{5}{10000} \div \frac{5}{10} = \frac{5}{10000} \times \frac{10}{5} = \frac{1}{1000} = .001.$

10. Divide .05 by .05. $.05 \div .05 = \frac{5}{100} \div \frac{5}{100} = 1.$

From the nature of decimals as shown in these examples we have the following **Principles**:

1. *The dividend contains as many decimal places as the divisor and quotient together.*

2. *There must be as many decimal places in the quotient as the number of places in the dividend exceeds the number in the divisor.*

Let it be required to divide 1.38483 by 60.21.

ILLUSTRATION.

```
60.21)1.38483(.023
      1 2042
        18063
        18063
```

EXPLANATION.—We divide as in integers, obtaining 23 as the result. We have now only to place the decimal point, and the work is done. Since the dividend has five decimal places, and the divisor two, the quotient should have three decimal places. As there are but two figures in the result, we supply the deficiency by prefixing a cipher, and place the decimal point at the left. Hence the following

RULE.

Divide as in integral numbers, and point off in the quotient, from the right hand, as many places for decimals as the number of decimal places in the dividend exceeds that in the divisor; and if there are not so many places, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide .74 by .25.

2. Divide 3.74 by .25.

3. Divide .008 by .5.

4. Divide 7.74 by 4.8.

5. Divide 2.56 by .0032.

6. Divide 3.1 by .25.

7. Divide 79.1 by .125.

8. Divide .12 by 1.6.

9. Divide .8 by 3.2.

10. Divide 8.25 by 25.

If the number of decimal places in the dividend is less than the number in the divisor, make them equal by annexing decimal ciphers to the dividend, and then divide.

11. Divide 6 by .003.

12. Divide 28.475 by .0005.

To divide by 10, 100, etc., we move the decimal point as many places to the left as there are ciphers in the divisor.

13. Divide 37.41 by 10.

14. Divide 341.6 by 100.

15. Divide 843.7 by 1000.

16. Divide 83.86 by 100.

TEST QUESTIONS.

If tenths are divided by tenths, how many decimal places will there be in the quotient? How many, if hundredths are divided by tenths? How many, if thousandths are divided by tenths? How many, if thousandths are divided by hundredths? Give the rule for division of decimals. State the three principles growing out of the nature of decimals. If the number of decimal orders in the dividend is less than those in the divisor, what must be done? How do you divide by 10, 100, 1000, etc.?

REDUCTIONS.

150. ***To reduce decimals to common fractions.***

Reduce .75 to a common fraction.

ILLUSTRATION.

$.75 = \frac{75}{100} = \frac{3}{4}$

EXPLANATION.—We write the decimal in the form of a common fraction; and reduce it to its lowest terms. Hence, decimals may be reduced to common fractions by the following

RULE.

Express the decimal in the form of a common fraction, and reduce it to its lowest terms.

EXAMPLES.

Reduce the following decimals to common fractions:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
.032.	3.112.	7.002.	70.03.	.00012.

151. ***To reduce common fractions to decimals.***

Reduce $\frac{3}{4}$ to a decimal fraction.

ILLUSTRATION.

$$4\overline{)3.00}$$
$$.75$$

EXPLANATION.—$\frac{3}{4}$ is 3 divided by 4. If we annex one decimal cipher to 3, we obtain $3.0 = \frac{30}{10}$. This we divide by 4, and obtain the quotient $\frac{7}{10}$ and a remainder $\frac{2}{10}$. We write the quotient under the dividend, annex a cipher to the remainder 2, making $\frac{20}{100}$; divide it by 4, and write the result $\frac{5}{100}$ in hundredths place in the quotient. Hence the

RULE.

Annex as many decimal ciphers to the numerator as may be desired; divide the result by the denominator and point off from the right of the quotient a number of decimal figures equal to the number of ciphers annexed.

EXAMPLES.

Reduce the following common fractions to decimals:

1. $\frac{7}{8}$. *3.* $\frac{3}{5}$. *5.* $\frac{5}{7}$. *7.* $\frac{17}{20}$.
2. $\frac{11}{16}$. *4.* $\frac{11}{25}$. *6.* $\frac{18}{23}$. *8.* $\frac{9}{11}$.

The exact value of some common fractions cannot be expressed in decimals, as in Example 5. In such cases, continue the division until you obtain three decimal figures in the quotient, and annex the sign + to the result to denote incompleteness.

Reduce the following fractions to decimals:

9. $\frac{7}{11}$. *11.* $\frac{4}{17}$. *13.* $18\frac{2}{13}$. *15.* $\frac{1}{2}$ of $\frac{3}{7}$.
10. $\frac{9}{13}$. *12.* $6\frac{3}{16}$. *14.* $10\frac{4}{11}$. *16.* $\frac{15}{17}$.

NOTE.—If the number of figures in the quotient is less than the number of ciphers annexed, prefix the requisite number of ciphers.

PRACTICAL PROBLEMS.

1. If 3.5 bushels of wheat cost $4.375, what does 1 bushel cost? What do 2 bushels cost?

2. If 3.7 barrels of apples cost $20.35, what does 1 barrel cost? What do $2\frac{1}{2}$ barrels cost?

3. If a boat sails 34.78 miles in 3.7 hours, how far does she sail in 1 hour? How far in 2 hours?

4. If 18.2 *lbs.* of butter cost $6.825, what does 1 pound cost? What do 2 *lbs.* cost? What 5 *lbs.*?

5. What is the sum of 4.1 *acres*, 3.72 *acres*, 4.82 *acres*, and 8.15 *acres*? $5.3 + 6.05 + 7.005 + 3.06 = ?$

6. If a man travels 34.2 miles the first day, 19.9 miles the second day, 18.87 miles the third day, and 19.7 miles the fourth day, how far does he travel altogether?

7. A grocer bought 15.4 *lbs.* of butter at 37.5 cents a pound, and sold the lot for $7.225; how much did he gain by the bargain?

8. A. and B. start from the same point and travel in opposite directions; A. travels at the rate of 17.4 miles a day, and B. at the rate of 16.9 miles a day; how far apart are they at the end of 4.7 days?

9. If 6.5 *lbs.* of sugar costs 74.75 cents, what does 1 *lb.* cost? What do 3.7 *lbs.* cost?

10. If two persons are 38 miles apart, and start toward each other, the first at the rate of 3.5 miles per hour, and the second at the rate of 4.1 miles per hour, how long before they will meet?

REVIEW QUESTIONS.

What is a decimal unit? What is a decimal fraction? Define a decimal point. In what two ways may a decimal be written? Explain the method of reading a decimal. Repeat the numeration table from units to millions, and from units to millionths. State the three principles of decimals. Give the rule for reducing decimal fractions to common fractions. Give the rule for reducing common fractions to decimals. Reduce $\frac{3}{8}$ to a decimal. What is the rule for addition of decimals? For subtraction of decimals? What is the rule for multiplication of decimals? For division of decimals?

NOTE.—For more extended and thorough treatment of decimal fractions, use Davies and Peck's "Complete Arithmetic."

CURRENCY.

DEFINITIONS.

152. Currency is anything that circulates as lawful money.

153. Money is the measure of the value of things, used as a medium of trade.

154. Specie or coin is metal stamped with a die to give it a legal value, and authorized by government to be used as money.

155. A mint is a place in which the coin is manufactured.

156. Paper money consists of bills and notes authorized by government to circulate as money.

157. A **Denominate Number** is one whose unit is named; as, 3 *pounds*, 4 *feet*, 5 *dollars*.

158. A **simple denominate number** is a unit or collection of units of the same denomination; as, 1 *lb.*, $3, 8 *ft.*, etc.

159. A **compound number** is a collection of units of the same nature, but of different denominations; as, 2 *feet and* 4 *inches*; 3 *dollars and* 5 *cents*; 5 *pounds and* 8 *ounces*.

160. The **scale of a compound number** is a succession of numbers showing how many times the unit of each denomination is contained in the unit of the next higher denomination. Thus, in the table of English

money, 4 farthings make 1 penny, 12 pence 1 shilling, 20 shillings 1 pound. The scale is 4, 12, 20. The scale of the United States currency is 10, 10, 10, 10, etc.

The first is *varying;* the second is *uniform.*

UNITED STATES CURRENCY.

161. The currency of the United States is a decimal currency.

TABLE.

10 mills (*m.*)	make	1 cent	*ct.*
10 cents	"	1 dime	*d.*
10 dimes	"	1 dollar	$.
10 dollars	"	1 eagle	*E.*

The coin of the United States consists of gold, silver, nickel and bronze.

GOLD.—The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

SILVER.—The trade-dollar, dollar, half-dollar, quarter-dollar, ten-cent piece, and five-cent piece.

NICKEL.—Five-cent and three-cent pieces.

BRONZE.—The one-cent and two-cent pieces.

EXERCISES FOR ORAL WORK.

1. How many mills in 1 cent? In 4 cents? In 8 cents?

2. How many cents in 1 dime? In 5 dimes? 6 dimes?

3. How many dimes in 1 dollar? In 8 dollars? In $10?

4. How many cents in 20 mills? In 30 mills? In 40?

5. How many dimes in 50 cents? In 80 *cts.*? In 90 *cts.*?

6. How many dollars in 40 dimes? In 60 dimes?

7. How many dimes and cents in 25 cents? In 75 *cts.*?

8. How many cents in 5 dollars? In 50 dollars?

9. How many dollars in 300 cents? In 2,000 cents?

10. How many cents in 15 dollars? In 18 dollars?

Expressions for United States currency are usually written in the form of a mixed decimal, the *dollar* being taken as the primary unit. Thus, the expression 3 *eagles,* 2 *dollars,* 5 *dimes,* 4 *cents,* and 3 *mills* is written $32.543, and read 32 *dollars,* 54 and 3 *tenths cents.*

The terms *dime* and *eagle* are but little used; the term *mill* is seldom employed, except in official reports and in laying taxes. In business operations mills are generally expressed as fractions of a cent. Since dimes are tenths of a dollar, when the number of cents is less than ten, we write a cipher in the place of tenths. Thus, $2.05.

Expressions for United States currency are usually read in dollars and cents.

Read

$31.25. $123.04. $108.036. $100.042. $88.888.

Application of the Rules for Decimals to U. S. Currency.

EXAMPLES FOR WRITTEN WORK.

1. Write decimally 3 *eagles,* 4 *dollars,* 3 *dimes,* 5 *cents,* and 2 *mills.*

2. Write 14 *eagles,* 4 *dimes,* 5 *cents,* and 7 *mills.*

3. Write decimally 86 *eagles*, 3 *dollars*, 2 *cents*, and 3 *m.*

4. Write decimally 70 *eagles*, 4 *dollars*, and 8 *cents.*

5. Add \$16.54, \$13.43, \$81.415, and \$9.607.

6. Add \$3.814, \$40.60, \$31.875, and \$118.436.

7. Add \$8.14, \$32.415, \$4.675, and \$31.843.

8. Add \$0.17, \$8.477, \$30.303, and \$8.888.

9. From \$14.62 subtract \$1.89. \$13.43—\$9.235 = ?

10. Find the difference between \$25.873 and \$19.984.

11. A man bought a box of raisins for \$3.375, a box of candles for \$4.62, and 75 *lbs.* of sugar for \$9.465; what did he pay for the whole?

12. A person bought a hat for \$4.75, a coat for \$19.65, a pair of boots for \$8.44, and an umbrella for \$1.69; what did they all cost him?

13. A bookseller sold a dictionary for \$8.25, 6 readers for \$7.44, 5 arithmetics for \$4.75, and 20 spellers for \$4.75; what did he receive for them all?

14. A dealer bought cloth for \$18.42, and sold it again for \$27.105; what did he gain?

15. A boy had \$4.75 at the beginning of the week, and earned during the week \$5.92; if he spends \$6.18, how much will he then have?

16. A farmer sold a horse for \$187.25, two cows for \$84.90, and 40 bushels of oats for \$22.46, for which he received a wagon worth \$213.26, and the balance in cash; how much cash did he receive?

17. Multiply \$4.75 by 9.

18. \$12.625 × 13 = ?

19. \$88.775 × 27 = ?

20. \$84.35 × 12.2 = ?

21. 118.26 ÷ 9 = ?

22. \$1,220.03 ÷ 14 = ?

23. \$2,022.02 ÷ \$77.77 = ?

24. \$2,895.0438 ÷ \$86.86 = ?

CANADA CURRENCY.

162. The currency of the Dominion of Canada is decimal; it is reckoned in *dollars* and *cents.*

The coin of Canada is silver and bronze.

FRENCH CURRENCY.

The coin of France is gold, silver and bronze.

TABLE.

10 centimes (*ct.*)	make	1 decime, *dc.*
10 decimes	"	1 franc, *fr.*

The primary unit is 1 franc; its value is \$0.193.

ENGLISH MONEY.

163. This is the national currency of Great Britain.

The primary unit is 1 *pound sterling.*

TABLE.

4 farthings (*far.*, or *qr.*)	make	1 penny, *d.*
12 pence	"	1 shilling, *s.*
20 shillings	"	1 pound sterling, £.
21 shillings	"	1 guinea, *g.*

The sign £ is usually written before the number to which it applies.

The coin of Great Britain is gold, silver and copper.

The pound is the unit of the *money of account*, and the coin representing it is called a sovereign.

The value of the sovereign is \$4.8665 U. S.

2 shillings make 1 florin, *fl.*

5 shillings make 1 crown, *cr.*

EXAMPLES.

1. In £5 how many shillings? How many in £8?

2. How many pence in 18*s.*? How many in 11*s.*?

3. How many pounds in 40*s.*? How many in 60*s.*?

4. How many dollars in 10 *sov.*? How many in 8 *sov.*?

5. How many *sov.* in 100*s.*? How many in 120*s*?

6. In 6 francs how many centimes?

7. In 5 francs how many decimes?

8. In 250 centimes how many francs?

9. In 140 francs how many U. S. dollars?

10. In $20 how many francs?

11. How many U. S. dollars in £4?

12. How many U. S. dollars in 60*s.*?

13. How many U. S. dollars will it require to pay a draft from London amounting to 258 pounds sterling?

14. I wish to pay a merchant in Paris 1250 francs; how many U. S. dollars will it require?

15. How many U. S. dollars and cents in £3 5*s.* 6*d.*

What is currency? What is money? What specie? What, a mint? Of what does paper money consist? What is the currency of the United States? Recite the table. How are expressions for U. S. currency usually written? When the number of cents is less than 10, what is written in place of tenths of cents? In what denominations are expressions of U. S. currency usually read? What is the currency of Canada? How reckoned? What is the coin of Canada? Of what does the coin of the U. S. consist? Name the denominations of the gold coin. Of the silver coin. Of nickel. Of bronze. What is the primary unit in the U. S. and Canada currency? Recite the table of French currency. What is the primary unit? What is its value in U. S. currency? Recite the table of English money. What is the primary unit? What is its value in U. S. currency? How is the pound sterling used? What coin takes its place in the currency of England? What is the value of the sovereign in U. S. currency?

METRIC SYSTEM.

164. The **Metric System** is a system of weights and measures based on a primary unit of length called a **Meter.** *The scale is decimal.*

The **Meter** is one *ten-millionth* part of the distance from the equator to the north pole. It is nearly equal to 39.37 inches.

MEASURE OF LENGTH.

165. The primary unit is the **meter.**

TABLE.

10 millimeters (*mm.*)	make	1 centimeter (*cm.*)
10 centimeters	"	1 decimeter (*dm.*)
10 decimeters	"	1 meter (*m.*)
10 meters	"	1 decameter (*Dm.*)
10 decameters	"	1 hectometer (*Hm.*)
10 hectometers	"	1 kilometer (*Km.*)
10 kilometers	"	1 myriameter (*Mm.*)

NOTE.—If possible show the class a *meter*, and explain its subdivisions. If none is at hand, construct one from the Decimeter on the 158th page.

EXERCISES AND EXAMPLES.

1. Measure your desk, and tell how many meters, or meters and decimeters long it is. How many wide?

2. Measure the door, both its height and width.

3. How many centimeters long is your arithmetic? How many wide?

4. How many milimeters wide is the Decimeter illustrated on this page?

5. How many decimeters in 12 meters?

6. How many centimeters in 20 millimeters? How many in 50?

7. Change 25 decameters to meters and decimals of a meter.

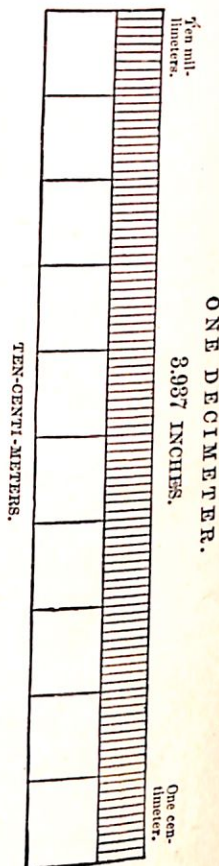
8. Change 225 hectometers to meters and decimals of a meter.

To Write Numbers in the Metric System.

Write the meters in the units place, decameters in tens place, hectometers in hundreds place, the kilometers in thousands place, myriameters in the ten thousands place, decimeters in the tenths place, centimeters in the hundredths place, and millimeters in the thousandths place.

To Read Numbers in the Metric System.

We may read a number in the Metric system in terms of all its units, or in terms of any one of them.

The expression 24.25 meters may be read 2 decameters, 4 meters, 2 decimeters, and 5 centimeters; but it is usually read 24 and *twenty-five hundredths* meters.

Exercises in Reading Numbers.

Read the following:

16 *m.*; 50 *dm.*; 75 *cm.*; 14 *mm.*; 17 *decam.*; 28 *kilom.*

MEASURE OF SURFACE.

166. The primary unit is the **Square Meter.**

TABLE.

100 sq. millimeters (*sq. mm.*) make 1 sq. centimeter, (*sq. cm.*)

100 sq. centimeters	make	1 sq. decimeter, (*sq. dm.*)
100 sq. decimeters	"	1 sq. meter, (*sq. m.*)
100 sq. meters	"	1 are, (*a.*)
100 ares	"	1 hectare, (*Ha.*)

For *land measure* the primary unit is the **are** (pronounced *ar*).

MEASURES OF VOLUME.

167. The primary unit is the **stere** (pronounced *stair*); it is a **cubic meter.**

TABLE.

1000 cu. centimeters (*cu. cm.*) make 1 cu. decimeter, (*cu. dm.*)

1000 cu. decimeters make 1 stere.

For Wood Measure.

10 decisteres (*d. st.*)	make	1 stere, (*st.*)
10 steres	"	1 decastere, (*Dst.*)

The primary unit is the stere = .2759 cords.

MEASURES OF CAPACITY.

The primary unit is the liter (*leeter*) = 61.026 cu. in.

TABLE.

10 centiliters (*cl.*)	make	1 deciliter, (*dl.*)
10 deciliters	"	1 liter, (*l.*)
10 liters	"	1 decaliter, (*Dl.*)
10 decaliters	"	1 hectoliter, (*Hl.*)
10 hectoliters	"	1 kiloliter, or stere, (*Kl.*)

MEASURE OF WEIGHTS.

168. The primary unit is the gram. It is the weight of a cubic centimeter of distilled water at 39° Fah.

TABLE.

10 milligrams (*mg.*)	make	1 centigram, (*cg.*)
10 centigrams	"	1 decigram, (*dg.*)
10 decigrams	"	1 gram, (*g.*)
10 grams	"	1 decagram, (*Dg.*)
10 decagrams	"	1 hectogram, (*Hg.*)
10 hectograms	"	1 kilogram, (*Kg.*)

Small weights are expressed in milligrams, and large ones in kilograms. The kilogram is the weight of a liter of water at 39° Fah.

EXAMPLES.

Change the following to meters:

(*1.*)	(*2.*)	(*3.*)	(*4.*)
30 *dm.*	800 *cm.*	12000 *mm.*	150 *Dm.*

(*5.*)	(*6.*)	(*7.*)
200 *Hm.*	5,000 *Km.*	30,000 *Mm.*

Change the following to square meters:

(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
200 *sq. dm.*	3000 *sq. cm.*	6000 *sq. mm.*	15 *a.*	27 *Ha.*

Reduce the following to steres:

(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)
5000 *cu. dm.*	15 *Dst.*	3000 *cu. dm.*	400 *dst.*	20 *Dst.*

Reduce the following to liters:

(*18.*)	(*19.*)	(*20.*)	(*21.*)	(*22.*)
500 *cl.*	120 *dl.*	20 *Dl.*	300 *Hl.*	4000 *Kl.*

Change

(*23.*)	(*24.*)	(*25.*)
20 *dg.* to *Dg.*	500 *Hg.* to *g.*	400 *mg.* to *cg.*

(*26.*)	(*27.*)
6,000 *g.* to *Kg.*	5 *Hg.* to *g.*

28. Express in kilograms and decimals of a kilogram,

3 *Kg.* 25 *Hg.* 15 *Dg.* 6 *g.*

On what is the Metric system based? What is the scale in the notation of the Metric system? The meter is what part of the distance from the equator to the north pole? What is its length in inches? Recite the table of length. The meter is $\frac{1}{10000000}$ part of what? How are numbers written in the Metric system? How may a number be read in the Metric system? Write the abbreviations of the denominations in the table of length. Recite the table of measure of surface. Draw a square decimeter on the blackboard. Draw a square centimeter. How many square millimeters in the square centimeter?

BUSINESS OPERATIONS.

TERMS USED IN BUSINESS TRANSACTIONS.

169. The cost of a thing is the *value in money* paid for it.

170. ***To find the cost of any number of things when we know the cost of one.***

1. What is the cost of 40 bushels of oats at 57 cents a bushel?

ILLUSTRATION.

$0.57
40
$22.80

EXPLANATION.—Since 1 bushel costs $0.57, 40 bushels will cost 40 times $0.57 = $22.80.

RULE.

Multiply the cost of one thing by the number of things.

EXAMPLES.

2. What is the cost of 5 yards of cloth at $5.50 a yard?

3. Find the cost of $12\frac{1}{2}$ cords of wood at $7.50 a cord.

4. Find the cost of 56 *lbs.* of pork at $9\frac{1}{4}$ *cts.* a pound.

Since it is not material which factor is used as the multiplier, in examples like the last, for the sake of convenience, we multiply by the smaller number, and observe the rule for pointing off decimals.

5. Find the cost of $32\frac{1}{2}$ *yds.* of ribbon at $37\frac{1}{2}$ *cts.* a yard.

6. Find the cost of $43\frac{1}{4}$ *lbs.* of tea at $92\frac{1}{2}$ *cts.* a pound.

7. A cask of wine contains $42\frac{3}{4}$ gallons, and is worth $2.13 per gallon; what is the value of the whole?

171. ***To find the cost of one thing, when we know the entire cost, and the number of things.***

1. If 50 oranges cost $2.25, what do they cost apiece?

Since 50 oranges cost $2.25, one orange costs $\frac{1}{50}$ of $2.25 = $0.045.

RULE.

Divide the entire cost by the number of things.

EXAMPLES.

2. Jane bought 47 *yds.* of calico for $6.34½; what did it cost per yard? *Ans.* $13\frac{1}{2}$ *cents.*

3. If $63\frac{1}{2}$ bushels of potatoes cost $35.24¼, what is the cost of 1 bushel, and what is the cost of 8 bushels?

4. Find the cost of 1 *lb.* of tea, when $37\frac{1}{2}$ *lbs.* cost $28.12½. Find the cost of 3 *lbs.*

5. If 44 *yds.* of linen cost $27.50, what is the cost of 1 *yd.*? Of $16\frac{1}{2}$ *yds.*? Of 26 *yds.*?

6. If 48 *lbs.* of grapes cost $15.36, what will 13 *lbs.* cost?

7. How many *lbs.* of sugar, at 14 *cts.* a pound, can be bought for 56 *lbs.* of butter, at $31\frac{1}{4}$ *cts.* per *lb.*?

8. A farmer sold 8 cows for $345; what was the average value of each?

172. ***To find the number of things when we know the entire cost, and the cost of one thing.***

1. How many pounds of sugar, at $12\frac{1}{2}$ cents a pound, can be bought for $106.25?

SOLUTION. $106.25 \div .12\frac{1}{2} = 850$, therefore 850 *lbs.* can be bought.

RULE.

Divide the entire cost by the cost of one thing.

EXAMPLES.

2. How many yards of muslin, at $19\frac{1}{2}$ cents a yard, can be bought for $22.815? How many for $50?

3. How many barrels of flour, at $12.44 a barrel, can be bought for $485.16? How many for $625?

4. If a man earns $18.75 per week, how many weeks will it take him to earn 323.43\frac{3}{4}$?

5. How many yards of linen, at $64\frac{1}{2}$ cents a yard, can be bought for $200.595? How many for $500?

6. How many barrels of flour, at $7.50 a barrel, can be bought for $217.50? How many for $750?

7. If tea costs $85\frac{1}{2}$ cents a pound, how many pounds can be bought for 117.56\frac{1}{4}$? How many for 235.12\frac{1}{2}$?

8. How many bushels of potatoes, at $37\frac{1}{2}$ cents a bushel, can be bought for 10.12\frac{1}{2}$? How many for 30.37\frac{1}{2}$?

173. ***To find the cost of any number of things, when we know the cost of 100 or 1,000 things.***

1. What is the cost of 217 bananas, at $2.15 per hundred?

ILLUSTRATION.

$$\frac{\$2.15}{100} = \$0.0215$$

```
$0.0215
    217
   1505
   215
  430
$4.6655
```

EXPLANATION.—Dividing $2.15 by 100, gives the cost of one banana; and multiplying the cost of one by 217, gives the cost of 217 bananas.

RULE.

Multiply the cost of 100 or 1,000 things by the number of things, and move the decimal point two places to the left when the cost of 100 is given, and three places when the cost of 1,000 is given.

EXAMPLES.

2. Find the cost of 1,622 bricks, at $6.25 per thousand.

3. What is the value of 874 *ft.* of scantling, at $3.50 per hundred?

4. Find the cost of 3,240 shingles, at $5 per thousand.

5. How much must I pay for 375 *lbs.* of beef, at $7.50 per hundred?

6. If oranges sell at $2.25 per hundred, how much must I pay for 288 oranges?

174. ***To find the cost of any number of pounds when we know the cost of a ton of 2,000 pounds.***

1. Find the cost of a load of coal weighing 2,400 pounds, at $8.64 a ton.

ILLUSTRATION.

```
2)8.64
  4.32
  2400
172800
 864
$10.368.00
```

EXPLANATION.—Since $8.64 is the cost of 2,000 pounds, $\frac{1}{2}$ of $8.64 or $4.32 is the cost of 1,000 pounds; 2400 pounds will cost $\frac{2400}{1000}$ times $4.32. We multiply by 2400, and move the decimal point three places to the left, which is equivalent to dividing the product by 1000.

RULE.

Multiply half the cost of 1 ton by the number of pounds, and move the decimal point three places to the left.

EXAMPLES.

2. What is the cost of 8,136 *lbs.* of coal, at $7 per ton?

3. Find the cost of 3,714 *lbs.* of steel. at $18.50 a ton.

4. Find the cost of 8,176 *lbs.* of straw, at $16 a ton.

5. Find the value of 8,488 *lbs.* of hay, at $14 per ton.

6. What is the cost of transportation of 3,635 *lbs.*, at the rate of $1.28 per ton?

7. Find the value of 70,432 *lbs.* of ore, at $2.50 per ton.

8. If building stone is worth $1 per ton, what will 88,311 *lbs.* cost?

ALIQUOT PARTS.

175. An **Aliquot Part** of a number is one of the equal parts, whether integral or fractional, into which the number can be divided.

The most important aliquot parts of a dollar are given in the following

TABLE.

50 *cts.* is $\frac{1}{2}$ of $1.	$12\frac{1}{2}$ *cts.* is $\frac{1}{8}$ of $1.
$33\frac{1}{3}$ *cts.* " $\frac{1}{3}$ of $1.	10 *cts.* " $\frac{1}{10}$ of $1.
25 *cts.* " $\frac{1}{4}$ of $1.	$6\frac{1}{4}$ *cts.* " $\frac{1}{16}$ of $1.
20 *cts.* " $\frac{1}{5}$ of $1.	5 *cts.* " $\frac{1}{20}$ of $1.

176. ***To find the cost of any number of things when 1 thing costs an aliquot part of $1.***

1. What is the cost of 60 *lbs.* of butter, at $33\frac{1}{3}$ *cts.* a pound?

ILLUSTRATION.—At $1 a pound, 60 pounds will cost $60. Since the cost of 1 pound is $\frac{1}{3}$ of a dollar, the cost of 60 pounds will be $\frac{1}{3}$ of $60 or $20.

RULE.

Divide the number of things by the number of times that the price of one thing is contained in $1.

EXAMPLES.

2. What is the cost of 40 bushels of potatoes, at $33\frac{1}{3}$ cents a bushel?

3. Find the cost of 87 *yds.* of sheeting, at 25 cents a *yd.*

4. What is the cost of 147 *yds.* of calico, at 20 *cts.* a yard? What is the cost of 180 yards?

5. How much will 168 *lbs.* of sugar come to, at $12\frac{1}{2}$ *cts.* a pound? How much at $33\frac{1}{3}$ cents?

6. Find the cost of 64 bushels of oats, at $37\frac{1}{2}$ *cts.* a bushel.

The cost will be 3 times as much as though the price was $12\frac{1}{2}$ cents a bushel; hence it is $\frac{1}{8}$ of 64×3 or $24.

7. Find the cost of 76 bushels of rye, at 75 *cts.* a bushel. Find the cost of 225 bushels.

8. How much will 118 *lbs.* of soap cost, at $12\frac{1}{2}$ *cts.* a pound? How much, at $6\frac{1}{4}$ cents?

177. ***To find the number of things when the entire cost is known, and the cost of one is an aliquot part of a dollar.***

1. How many bushels of apples at $0.50 a bushel, can be bought for $212?

ILLUSTRATION.

$\$212 \div \$\frac{1}{2} = 424$

EXPLANATION.—Since $\$\frac{1}{2}$ will buy one bushel, $212 will buy as many bushels as there are $\$\frac{1}{2}$ in $212.

RULE.

Divide the entire cost by such a fraction as will express the cost of one as an aliquot part of a dollar.

EXAMPLES.

2. How many baskets of pears can be bought for $10 at $33\frac{1}{3}$ cents a basket?

3. How many pineapples can be bought for $22 at $12\frac{1}{2}$ cents each?

4. How many pounds of cheese can be bought for $27 at $0.25 a pound? How many for $50?

5. How many quarts of peas can be bought for $3 at $0.20 a quart? How many for $5?

6. How many bushels of oats, at 75 cents a bushel, can be bought for 45 bushels of wheat at $2.25 a bushel?

What is currency? Illustrate. Repeat the table of United States currency. In what terms do we write United States currency? By what rules do we operate on United States currency? How do you find the cost of any number of things when you know the cost of one? What is cost? How do you find the cost of one when you know the entire cost and the number of things? How do you find the number of things when you know the entire cost, and the cost of one? How do you find the cost of any number of things sold by the hundred or thousand? How do you find the cost of any number of pounds when you know the price per ton? What is an aliquot part of a number? How do you find the cost of any number of articles when the price of one article is an aliquot part of a dollar?

BILLS AND ACCOUNTS.

178. A **Bill** is a written statement of goods sold, of services rendered, or of money paid.

179. An **Account** is a written statement of items of debit and credit.

180. A **Debtor** is a person who owes another.

181. A **Creditor** is a person to whom something is owed.

182. A **Bill is receipted** when the words "Received Payment" are written at the bottom, and the creditor's name is lawfully signed.

183. A **Footing** is the sum or amount of a bill.

184. **Credit** is what the creditor pays, or has paid. **Debit** is what the debtor owes.

185. The **Balance** is the difference between the footings of the debit and credit sides of an account.

186. **Abbreviations used in Accounts.**

@. At.	Doz. Dozen.
% Account.	Dr. Debit or Debtor.
Am't. Amount.	No. Number.
Bal. Balance.	Pd. Paid.
Co. Company.	Per. By.
Cr. Credit or Creditor.	Rec'd. Received.
Do. The Same.	% Per cent.

187. ***To find the footing of a bill.***

Find the amount of each item and take the sum of the results.

Find the amount of the following bill of items:

NEW YORK, May 1, 1878.

Mr. James Spendthrift,

Bought of Benjamin Saveall,

Item	$	cts.
16 pounds of Tea, @ $0.85 per pound		
20 " " Coffee, @ $0.15½ per pound		
15 yards of Linen, @ $0.66 per yard		
Amount, ...	$	

Received payment,

BENJAMIN SAVEALL.

ALBANY, June 2, 1878.

Mr. Jacob Johns,

Bought of Gideon Gould,

Item	$	cts.
36 pounds of Sugar, at 9½ cents per pound . . .		
3 hogsheads of Molasses, 63 galls. each, @ 27 cents a gallon		
5 casks of Rice, 200 pounds each, @ 5 cts. per pound		
2 chests of Tea, 80 pounds each, @ 96 cts. per pound		
Total cost, . . .	$	

Received payment, for GIDEON GOULD,

CHARLES CLARK.

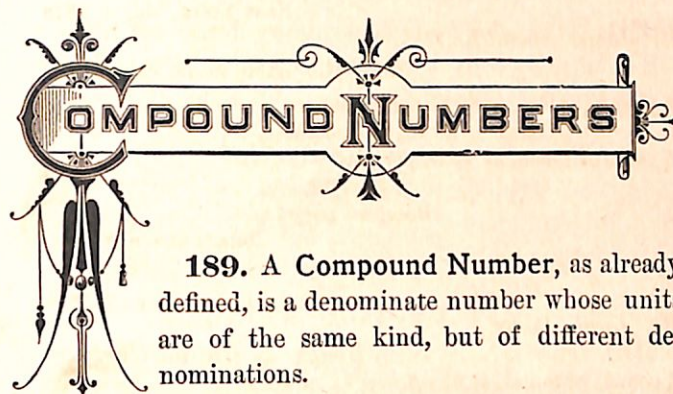
188. *To find the balance of an account.*

Find the difference of the debit and credit items of the account.

Find the balance of the following account:

A. T. Stewart & Co.

In account with A. L. Compton.

DR.					CR.				
1878									
Jan.	12	To 200 lbs. Butter @ $.30			Feb.	5	By 20 yds. Cloth @ $2.		
Feb.	10	" 900 lbs. Cheese " .18			Mar.	8	" 25 prs, Gloves " 1.20		
"	15	" 400 lbs. Lard " .12			Apr.	1	" 12 yds. Linen " 0.50		
Mar.	3	" 500 lbs. Tallow " .16							
Apr.	4	" 80 doz. Eggs " .25			Apr.	4	Balance		

189. A **Compound Number,** as already defined, is a denominate number whose units are of the same kind, but of different denominations.

Denominate numbers are of the *same kind* when they can be expressed in terms of a common unit. Thus, 3 *shillings* and 5 *pence* are of the same kind, because they can both be expressed in *pence.*

If two denominate numbers are of the same kind, that which has the *greater unit* is said to be of the *higher denomination.* Thus, 3 shillings is of a higher denomination than 5 pence.

TABLES OF WEIGHTS.

TROY WEIGHT.

190. This is used in weighing gold, silver, and some kinds of precious stones.

The primary unit is 1 pound.

TABLE.

24 grains (*gr.*)	make	1 pennyweight .	*dwt.*
20 pennyweights	"	1 ounce . . .	*oz.*
12 ounces	"	1 pound Troy .	*lb. Tr.*

APOTHECARIES WEIGHT.

191. This table is used in mixing medicines.

The primary unit is 1 pound, the same as Troy weight.

TABLE.

20 grains (*gr.*)	make	1 scruple . .	℈
3 scruples	"	1 dram . . .	ʒ
8 drams	"	1 ounce . . .	℥
12 ounces	"	1 pound . .	℔

AVOIRDUPOIS WEIGHT.

192. This is used in weighing the ordinary articles of trade and commerce.

The primary unit is 1 pound, equal to 7000 grains Troy weight.

TABLE.

16 ounces (*oz.*)	make	1 pound . . .	*lb.*
25 pounds	"	1 quarter . . .	*qr.*
4 quarters	"	1 hundred . .	*cwt.*
20 hundred	"	1 ton	*T.*

NOTE.—In weighing coarse articles liable to wastage, as coal at the mines, etc., it is customary to call 112 lbs. a hundredweight, and 28 lbs. a quarter.

TABLE OF TIME.

193. Time is a measured portion of duration. Its primary unit is one solar day.

An **astronomical year** is the time required for the earth to perform one revolution round the sun; but this period does not contain an exact number of days; hence, for civil purposes, an artificial year is adopted. The artificial or civil year has sometimes *365 days*, and sometimes *366 days*, so distributed that, after a long period, the average length of the civil year is nearly equal to that of the astronomical year.

Every year divisible by 4 (except *centennial* years not divisible by 400) are leap years, all other years are common years.

TABLE.

60 seconds (*sec.*)	make	1 minute . . .	*min.*
60 minutes	"	1 hour	*hr.*
24 hours	"	1 day	*da.*
7 days	"	1 week	*wk.*
365 days	"	1 common year .	*c. yr.*
366 days	"	1 leap year . .	*l. yr.*
100 years	"	1 century . . .	*C.*

The civil year is divided into 12 unequal parts called months. Their order, and the number of days in each, are shown in the table following:

TABLE OF MONTHS.

1. January . .	31 days,	7. July	30 days,
2. February .	28 "	8. August . . .	31 "
3. March . . .	31 "	9. September . .	30 "
4. April	30 "	10. October . . .	31 "
5. May	31 "	11. November . .	30 "
6. June	30 "	12. December . .	31 "

February has 28 days in common years, and 29 days in leap years.

MEASURES OF LENGTH.

194. Magnitude is anything that can be measured.

195. A **Line** is the magnitude of length without regard to thickness or breadth.

A curved line is one whose direction changes at every point; as GH.

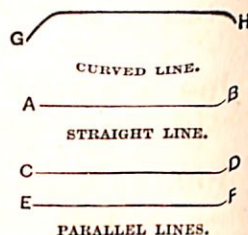
A straight line is one whose direction does not change at any point; as AB.

Straight lines are parallel when they have the same direction; as CD and EF.

The length of a line is the number of times it contains a given straight line taken as a unit.

CURVED LINE.

STRAIGHT LINE.

PARALLEL LINES.

LONG MEASURE.

196. This table is used in measuring distances and lines. The yard is the primary unit.

1	2	3

TABLE.

12 inches (*in.*)	make	1 foot . . .	*ft.*
3 feet	"	1 yard . . .	*yd.*
$5\frac{1}{2}$ yards	"	1 rod . . .	*rd.*
40 rods	"	1 furlong . .	*fur.*
8 furlongs, or 320 rods	"	1 mile . . .	*mi.*
3 miles	"	1 league . .	*lea.*

SURVEYORS' MEASURE.

197. Used in measuring land.

The unit is a Gunter's chain which is equal to 66 feet. It is divided into 100 equal parts called links.

TABLE.

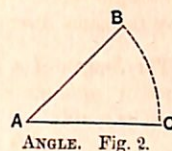
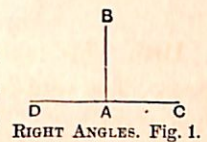
7.92 inches	make	1 link . .	*li.*
100 links, or 4 rods	"	1 chain . .	*ch.*
80 chains, or 320 rods	"	1 mile . .	*mi.*

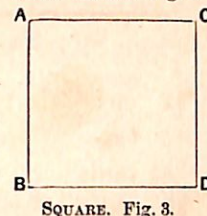
MEASURES OF SURFACE.

DEFINITIONS.

198. A **Surface** is a magnitude that has length and breadth, without thickness.

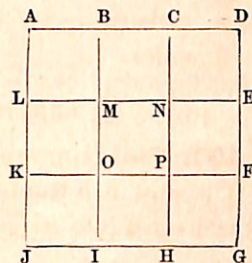
199. An **Angle** is the opening between two lines that meet at a point; as, BAC. The lines AB and AC are called **sides**, and the point A is called the **vertex** of the angle.

ANGLE. Fig. 2.

If the angle BAD is equal to the angle BAC, BA is **perpendicular** to DC, and the angles BAD and BAC are **right angles.**

RIGHT ANGLES. Fig. 1.

200. A **Square** is a plane figure bounded by four equal sides, and whose angles are right angles; as Fig. 3.

SQUARE. Fig. 3.

201. A **Rectangle** is a plane figure whose opposite sides are equal, and whose angles are right angles.

A **square foot** is a square whose sides are each equal to one foot; a **square yard** is a square whose sides are each equal to 1 yard.

SQUARE. Fig. 4.

The unit of measure of a surface is a square, one of whose sides is equal to the unit of length.

202. The **Area** of a surface is an expression for that surface in terms of a square unit. Figure 4 represents a square yard, each side of which is three feet long; the **area** is nine square feet.

The area of a square or rectangle is found by multiplying its length by its breadth.

SQUARE MEASURE.

Used for measuring surfaces.

TABLE.

144	square inches (*sq. in.*)	make	1 square foot	*sq. ft.*
9	square feet	"	1 square yard	*sq. yd.*
$30\frac{1}{4}$	square yards	"	1 square rod	*sq. rod.*
160	square rods	"	1 acre	*A.*

LAND MEASURE.

This is used in finding the area of land.

TABLE.

10,000	square links (*sq. li.*)	make	1 square chain	*sq. ch.*
10	square chains	"	1 acre	*A.*
640	acres	"	1 square mile	*sq. mi.*

In government surveys, a square mile is called a *section*, and 36 sections make a *township*.

MEASURE OF VOLUME AND CAPACITY.

203. A **volume** or **solid** is a magnitude that has *length*, *breadth*, and *thickness* or *height*.

204. A **cube** is a volume or solid bounded by six equal squares. The bounding squares are called **faces.** The sides of the squares are called **edges** of the cube.

CUBE.

205. A **unit of volume** is a cube whose edges are equal to a unit of length.

PARALLELOPIPEDON.

CUBIC MEASURE.

206. This is used in measuring volumes or solids.

TABLE.

1728	cubic inches (*cu. in.*)	make	1 cubic foot	*cu. ft.*
27	cubic feet	"	1 cubic yard	*cu. yd.*

A **cord of wood** is a pile 4 ft. wide, 4 ft. high, and 8 ft. long. A foot in length from such a pile is a **cord foot.**

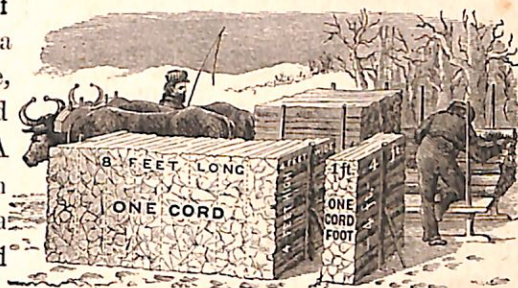



TABLE.

16	cubic feet (*cu. ft.*)	make	1 cord foot	*C. ft.*
8	cord feet	"	1 cord	*C.*

DRY MEASURE.

207. This is used in measuring grain, fruit, salt, etc.

The primary unit is 1 bushel. It is a cylindrical measure $18\frac{1}{2}$ inches across, and 8 inches deep; containing $2,150\frac{2}{5}$ cubic inches.

TABLE.

2 pints (*pt.*)	make	1 quart	*qt.*
8 quarts	"	1 peck	*pk.*
4 pecks	"	1 bushel	*bu.*

LIQUID MEASURE.

208. This is used in measuring liquids.

The primary unit is 1 gallon. It contains 231 cubic inches.

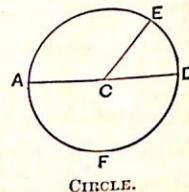
TABLE.

4 gills (*gi.*)	make	1 pint	*pt.*
2 pints	"	1 quart	*qt.*
4 quarts	"	1 gallon . . .	*gal.*
$31\frac{1}{2}$ gallons	"	1 barrel . . .	*bbl.*
2 barrels	"	1 hogshead . .	*hhd.*

ANGULAR MEASURE AND LONGITUDE.

DEFINITIONS.

209. A **Circle** is a portion of a plane bounded by a curved line every point of which is equally distant from a point within called the *centre;* as, AEDF. The bounding line is called the *circumference;* as, AEDF. Any part of the bounding line is called an *arc;* as, ED.

CIRCLE.

If a circumference is divided into 360 equal parts, each part is called a *degree.*

210. A **Diameter** is a straight line passing through the centre and terminating in the circumference; as, AD.

211. A **Radius** is a straight line drawn from the centre to any part of the circumference; as, CD, CE.

ANGULAR MEASURE.

212. The primary unit is the right angle. The ninetieth part of a right angle is a degree.

TABLE.

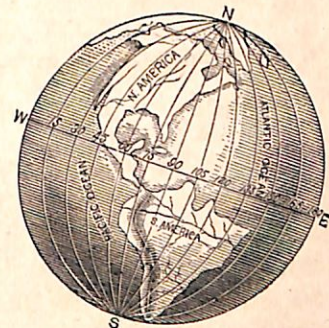
60 seconds (″)	make	1 minute . . .	′.
60 minutes	"	1 degree . . .	°.
90 degrees	"	1 right angle . .	*rt. a.*
360 degrees	"	1 circumference .	*cir.*

LONGITUDE.

213. Longitude of a place is the distance east or west from a given meridian.

The earth revolves on its axis once in 24 hours.

The circumference of the earth is a circle, and, like other circles, is supposed to be divided into 360 equal parts called degrees (°).

The sun appears to pass

MERIDIAN LINES.

entirely round the earth, 360°, once in 24 hours; in 1 hour it passes $\frac{1}{24}$ of 360°, or 15° of longitude. Hence, for a difference of 15° of longitude there is a difference of 1 hour of time, for 1° of longitude there is a difference of $\frac{1}{15}$ of 1 hour of time, or 4 minutes; for 15 of longitude there is a difference of 1 minute of time; for 15″ of longitude there is a difference of 1 second of time. It is noon, or 12 o'clock, at any place when the sun is on the meridian of that place; 15° west of that place it is 1 hour earlier, or 11 o'clock; 15° east it is 1 hour later, or 1 o'clock, etc. Hence the following

TABLE.

A difference of 15° of longitude makes a difference of 1 hour of time.

A difference of 15′ of longitude makes a difference of 1 minute of time.

A difference of 15″ of longitude makes a difference of 1 second of time.

REDUCTION.

214. Reduction is the process of changing the form of a number without altering its value. Thus, $25 may be expressed as 2500 cents, 500 cents may be expressed as $5.

215. Reduction Descending is the operation of changing a number from a higher to a lower denomination; as, dollars to cents.

216. Reduction Ascending is the operation of changing a number from a lower to a higher denomination; as, cents to dollars.

EXAMPLES IN REDUCTION DESCENDING.

FOR ORAL WORK.

217. *1.* How many feet in 3 yards? In 5 yards?

2. How many feet in 4 yards and 2 feet? 5 *yds.* 3 *ft.*?

3. Reduce 16 feet to inches. Reduce 15 yards to *in.*

4. Reduce 2 miles to chains. Reduce 5 miles to chains.

5. Reduce $2\frac{1}{2}$ *sq. yds.* to *sq. ft.* Reduce 5 *sq. ft.* to *sq. in.*

6. How many square chains in 12 acres? In 15 acres?

7. How many quarts in 4 gallons and 1 quart?

8. In 5 bushels and 3 pecks how many pecks?

9. In 4 *lbs.* Avoirdupois how many ounces?

10. How many scruples in 6 *drs.*? In 12 *drs.*?

11. How many pecks in 3 bushels of wheat?

12. How many minutes in 6 hours and 10 minutes?

13. How many pints in 7 quarts? In 10 *qts.*?

14. How many pence in 3 crowns? In $8\frac{1}{2}$ crowns?

15. How many degrees in 2 right angles? In 4 right angles?

EXAMPLES FOR WRITTEN WORK.

Reduce £18 14*s.* 9*d.* 3 *far.* to farthings.

ILLUSTRATION.

```
£18  14s.  9d.  3 far.
 20
----
 374
  12
----
4497
   4
-----
17991 far.
```

EXPLANATION.—Since £1 is equal to 20 shillings, there are 20 times as many shillings as pounds, to which we add 14*s.* (18 × 20) + 14 = 374*s.* Since 1 shilling is equal to 12 pence, there are 12 times as many pence as shillings: to this we add 9 pence. (374 × 12) + 9 = 4,497*d.* Since 1 penny is equal to 4 farthings, there are 4 times as many farthings as pence, to which add 3 farthings. (4,497 × 4) + 3 = 17,991 *far.* Hence the following rule.

RULE.

I. Multiply the units of the highest denomination by the number of the scale that connects this denomination with the one next lower, and to the product add the units of the latter denomination.

II. Multiply this result by the number that connects it with the next lower denomination, and to the product add the units of that denomination.

III. Continue this operation till the required denomination is reached.

EXAMPLES.

1. Reduce 3 *lbs.* 4 *oz.* 5 *dwts.* Troy, to pennyweights.
2. Reduce 1 *lb.* 7 *oz.* 15 *dwts.* 4 *grs.* Troy, to grains.
3. Reduce 7 ʒ 2 ℈ 15 *grs.* to grains.
4. Reduce 4 ℥ 5 ʒ 2 ℈ to grains.
5. Reduce 3 *qrs.* 15 *lbs.* 4 *oz.* avoirdupois, to ounces.
6. Reduce 6 *T.* 10 *cwt.* 3 *qrs.* 4 *lbs.* to pounds.
7. Reduce 7 *wks.* 2 *da.* 4 *hrs.* to hours.
8. Reduce 4 *da.* 16 *hrs.* 42 *min.* to minutes.
9. Reduce 2 *wks.* 5 *da.* 8 *hrs.* to seconds.
10. Reduce 16 *rds.* 4 *yds.* 2 *ft.* 9 *in.* to inches.
11. Reduce 1 *mi.* 240 *rds.* 2 *yds.* to feet.
12. How many inches in 7 *ch.* 74 *li.*?
13. Reduce 4 *mi.* 3 *ch.* to links.
14. Reduce 40 *sq. rds.* 8 *sq. yds.* to square feet.
15. How many acres in 56 *sq. mi.*?
16. Reduce 2 *A.* 74 *sq. li.* to square links.
17. Reduce 20 *cu. ft.* 168 *cu. in.* to cubic inches.
18. How many pints in 4 *bu.* 3 *pks.* 6 *qts.*?
19. Reduce 5 *bu.* 2 *pks.* 4 *qts.* 1 *pt.* to pints.

EXAMPLES IN REDUCTION ASCENDING.

FOR ORAL WORK.

218. *1.* How many *ft.* in 216 *in.*? How many *yds.*?
2. How many dollars in 6400 cents? In 9600 cents?
3. In 54 square feet, how many square yards?
4. In 450 square chains, how many acres?
5. How many cubic yards in 162 cubic feet?
6. In 200 cord feet, how many cords of wood?
7. Reduce 360 square chains to acres.
8. How many hands in 164 inches?
9. Reduce 224 pints to pecks.
10. Reduce 92 quarts to gallons.
11. In 96 ounces, how many pounds Troy?
12. Reduce 48 ℈ to ounces?
13. Reduce 480 *dwt.* to pounds.
14. Reduce 1200 minutes to hours.
15. How many degrees are 480 minutes?
16. Reduce 240° of longitude to hours.
17. How many minutes are 480 seconds?
18. How many dozen oranges are 480 oranges?

FOR WRITTEN WORK.

1. Reduce 8,743*d.* to pounds sterling.

ILLUSTRATION.

12) 8743
20) 728*s.* + 7*d.*
£36 + 8*s.*

EXPLANATION.—Since 12*d.* make 1*s.*, there are $\frac{1}{12}$ as many shillings as pence. Hence, 8,743*d.* = 728*s.*; the remainder 7 we express in pence. Since 20*s.* make £1, there are $\frac{1}{20}$ as many pounds as shillings = £36; the remainder 8 we express as shillings; hence, 8,743*d.* = £36 8*s.* 7*d.*

RULE.

I. Divide the given number by the number of the scale that connects it with the next higher denomination; the remainder will be units of the same denomination as the dividend.

II. Divide the quotient by the number that connects it with the next higher denomination; the remainder will be units of the denomination of the new dividend.

III. Continue the operation till the required denomination is reached.

2. Reduce 11,911 grains to pounds Troy.
Ans. 2 *lbs.* 16 *dwts.* 7 *grs.*

3. Reduce 876 *dwts.* to pounds Troy.

4. Reduce 1,511 *grs.* to ounces, apothecaries' weight.

5. Reduce 1,594 *oz.* avoirdupois, to quarters.

6. Reduce 8,842 *lbs.* to tons. Reduce 4,412 *lbs.* to *tons.*

7. Reduce 7,620 minutes to days.

8. Reduce 687 *hrs.* to weeks. Reduce 348 *hrs.* to *wks.*

9. How many yards in 786 *in.*? In 1572 *in.*?

10. How many miles in 897 *rds.*? In 1794 *rds.*

11. Reduce 2,875 *in.* to yards. Reduce 575 *in.* to *yds.*

12. Reduce 510 *pts.* to bushels. Reduce 255 *pts.* to *bu.*

13. Reduce 352 *qts.* to bushels. Reduce 704 *qts.* to *bu.*

14. How many gallons in 840 *pts.*? In 420 *pts.*

15. Reduce 8,532 *gi.* to gallons. Reduce 2,133 *gi.* to *gal.*

16. How many degrees in 8,844″? In 4,422″?

17. How many cubic yards in 162 cubic feet?

18. In 200 cord feet how many cords of wood?

19. Reduce 360 square chains to acres.

ADDITION OF COMPOUND NUMBERS.

219. Addition of Compound Numbers is the operation of finding the sum of two or more compound numbers.

The principles are the same as in simple numbers.

Exercises for Oral Work.

1. What is the sum of 9 inches and 11 inches? Of 22 *in.* and 13 *in.*? Of 8 *ft.* and 17 *ft.*? Of 9*d.* and 16*d.*

2. What is the sum of 2 *ft.* 4 *in.*, and 6 *ft.* 3 *in.*?

3. What is the sum of 2 *yds.* 1 *ft.*, and 3 *yds.* 4 *ft.*?

4. What is the sum of 2 *gals.* 3 *qts.*, and 3 *gals.* 3 *qts.*?

5. What is the sum of 3*s.* 8*d.*, and 5*s.* 9*d.*?

6. What is the sum of 3 *yds.* 3 *ft.*, 5 *yds.* 1 *ft.*, and 7 *yds.*?

7. What is the sum of 2 *wks.* 3 *das.*, and 3 *wks.* 4 *das.*?

8. What is the sum of 8 *sq. ft.* 16 *sq. in.*, and 5 *sq. ft.*? 4 *sq. in.*?

9. Find the sum of 3 *bush.* 1 *pk.*, and 4 *bush.* 2 *pks.*

10. What is the sum of 2° 3′ 4″, and 3° 1′ 2″, and 5° 2′ 3″?

Operation of Addition of Compound Numbers.

Let it be required to find the sum of £7 4*s.* 3*d.*, £11 9*s.* 8*d.* and £14 12*s.* 9*d.*

ILLUSTRATION.

£	*s.*	*d.*
7	4	3
11	9	8
14	12	9
£33	6*s.*	8*d.*

Explanation.—We write the numbers so that units of the same denomination shall stand in the same column. We add the right hand column, and find its sum is 20*d.* = 1*s.* 8*d.*; we write 8*d.* under the column in the place of the amount, and add 1*s.* with the column of shillings, which amounts to 26*s.* = £1 6*s.*; we write 6*s.* under the column, and add £1 with the column of £s, which amounts to £33. The required sum is therefore £33 6*s.* 8*d.* Hence, the following

RULE.

I. Write the numbers so that units of the same denomination shall stand in the same column.

II. Add the units of the lowest denomination, and divide their sum by the number of the scale that connects this denomination with the next higher one; set down the remainder, and carry the quotient to the next column.

III. Add the units of the second column thus increased, and proceed as before, continuing the operation till all the columns have been added.

EXAMPLES.

Perform the following indicated additions:

(*1.*)

£	*s.*	*d.*
1	14	8
3	12	7
5	2	9

(*2.*)

ʒ	℈	*grs.*
1	2	10
2	1	9
3	1	7

(*3.*)

cwt.	*qrs.*	*lbs.*
4	1	18
2	3	7
11	2	13

(*4.*)

wks.	*da.*	*hrs.*
2	5	7
3	4	11
5	6	14
1	4	20

(*5.*)

yds.	*ft.*	*in.*
2	2	8
1	1	7
3	1	6
4	0	11

(*6.*)

°	′	″
20	15	20
18	13	15
21	14	30
16	30	10

(*7.*)

T.	*cwt.*	*qrs.*
2	1	15
1	3	20
3	2	18
5	3	7
1	2	23

(*8.*)

oz.	*dwts.*	*grs.*
11	19	12
4	16	5
2	11	15
6	3	7
1	15	4

(*9.*)

gals.	*qts.*	*pts.*
4	3	1
3	2	0
7	1	0
6	2	1
2	0	1

(*10.*)

bu.	*pks.*	*qts.*
5	3	4
7	1	5
11	0	3
3	2	6
6	3	1

What is addition of compound numbers? What compound numbers can be added? Give the rule for addition of compound numbers.

SUBTRACTION OF COMPOUND NUMBERS.

220. Subtraction of Compound Numbers is the operation of finding the difference between two compound numbers.

The principles are the same as in subtraction of simple numbers.

EXERCISES FOR ORAL WORK.

1. From 11 *in.* subtract 6 *in.* From 12 *ft.* subtract 9 *ft.*
2. What is the difference of 7 *pks.* and 4 *pks.*?
3. What is the difference between £3 4*s.*, and £1 2*s.*?
4. What is the difference between 2*s.* 7*d.*, and 1*s.* 4*d.*?
5. 8 *yds.* 2 *ft.* — 6 *yds.* 1 *ft.* = ?
6. 7 *gals.* 1 *qt.* — 5 *gals.* 3 *qts.* = ?
7. 15 *ft.* 8 *in.* — 9 *ft.* 11 *in.* = ?

OPERATIONS OF SUBTRACTION OF COMPOUND NUMBERS.

Let it be required to find the difference between £9 4*s.* 3*d.* and £2 18*s.* 6*d.*

ILLUSTRATION.

£	s.	d.
9	4	3
2	18	6
£6	5s.	9d.

EXPLANATION.—We write the subtrahend under the minuend, so that units of the same denomination shall stand in the same column. We begin at the right. Since 6d. is greater than 3d. we add 12d. to 3d. making 15d., from which we subtract 6d., and write the remainder 9d. beneath. To compensate for 12d. added to the minuend, we add 1s. equal to 12d. to the next number of the subtrahend, obtaining 19s., which being greater than 4s., we add 20s. to 4s., obtaining 24s., from which we subtract 19s., and write the remainder 5s. beneath. To compensate for 20s. added to the minuend, we add £1 equal to 20s. to the subtrahend, obtaining £3, which we subtract from £9, and write the remainder £6 beneath; hence, the remainder is £6 5s. 9d.

In like manner we treat all similar cases; hence, the following

RULE.

I. Write the subtrahend under the minuend so that units of the same denomination shall stand in the same column.

II. Subtract each number in the subtrahend from the one above it in the minuend, and write the remainder in the line below.

III. If any number in the subtrahend is greater than the one above it in the minuend, add to the latter as many units as make one of the next higher denomination, perform the subtraction, and add 1 to the next number in the subtrahend.

EXAMPLES.

	(1.)			(2.)			(3.)			(4.)		
	£	s.	d.	da.	hrs.	min.	yds.	ft.	in.	oz.	wts.	grs.
From	9	5	2	57	21	43	46	1	7	13	18	5
Subtract	6	3	1	49	23	39	35	2	10	9	19	23
Remainder,	3	2	1	7	22	4	10	1	9	3	18	6

	(5.)					(6.)					
	tun.	hhd.	gal.	qt.	pt.	yr.	wk.	da.	hr.	min.	sec.
From	151	3	50	3	2	95	25	4	20	45	50
Take	27	2	54	3	2	80	30	6	23	46	56

INTERVAL BETWEEN DATES.

To find the interval between two dates, we subtract the earlier from the later. To do this, we write the number of the year, the number of the month, and the day of the month of each date; we then perform the subtraction, counting 30 days to a month, and 12 months to a year.

ILLUSTRATION.

Dates.		*yrs.*	*mos.*	*da.*
June 3, 1776, written	. .	1776	6	3
Oct. 16, 1771, "	. .	1771	10	16
Interval	. . .	4 *yrs.*	7 *mos.*	17 *da.*

EXPLANATION.—Having written the dates as shown on the right, we proceed according to the preceding rule.

8. What is the interval between Nov. 10, 1862, and March 4, 1875?

9. What is the interval between Sept. 21, 1851, and Feb. 11, 1873?

MULTIPLICATION OF COMPOUND NUMBERS.

221. Multiplication of Compound Numbers is the operation of taking a compound number as many times as there are units in the multiplier.

The principles are the same as in the multiplication of simple numbers.

Exercises for Oral Work.

1. How many inches are 7 times 8 inches?

2. What is the product of 8*d.* multiplied by 11? Express the answer in shillings and pence; thus

8*d.* × 11 = 88*d.* = 4*s.* 8*d.* *Ans.*

3. What is the product of 2 *ft.* multiplied by 16? Express the answer in *yds.* and *ft.*

4. What is the product of 7 *qts.* multiplied by 13?

5. What is the product of 1 *ft.* 3 *in.* multiplied by 9?

6. What is the product of 4 *bu.* 3 *pks.* multiplied by 9?

7. What is the product of 1*s.* 7*d.* multiplied by 8?

8. What is the product of 8 *gals.* 2 *qts.* multiplied by 4?

9. Find the product of 2 *tons* 3 *cwt.* multiplied by 6?

10. What is the product of 2° 3′ 1″ multiplied by 5?

Operation of Multiplication of Compound Numbers.

Let it be required to multiply £4 2*s.* 5*d.* by 16.

ILLUSTRATION.

£	*s.*	*d.*
4	2	5
		16
£65	18*s.*	8*d.*

EXPLANATION.—We write the multiplier under the multiplicand, then multiply 5*d.* by 16, obtaining 80*d.* = 6*s.* 8*d.*; we write 8*d.* beneath, and add 6*s.* to the product of 2*s.* × 16, obtaining 38*s.* = £1 18*s.*; we write 18*s.* beneath and add £1 to the product £4 × 16, obtaining £65, which we write beneath; hence the following

RULE.

I. Multiply the units of the lowest denomination in the multiplicand by the multiplier, and divide the product by the number of the scale that connects this denomination with the one next higher; set down the remainder.

II. Multiply the units of the next higher denomination in the multiplicand by the multiplier, add the quotient to the product, and divide the sum by the number of the scale that connects this denomination with the one next higher: set down the remainder and preserve the quotient to be added to the next product.

III. Continue this operation till all the parts of the multiplicand have been multiplied.

EXAMPLES.

Perform the following multiplications:

(*1.*)

bu.	*pks.*	*qts.*
3	2	7
		11
40	3	5

(*2.*)

gals.	*qts.*	*pts.*
5	2	1
		12
67	2	0

(*3.*)

£	*s.*	*d.*
5	4	9
		7
36	13	3

(*4.*)

T.	*cwt.*	*lbs.*	*oz.*
2	14	10	12
			15
40	11	61	4

(*5.*)

wk.	*da.*	*hrs.*	*min.*	*sec.*
2	4	7	15	18
				12
31	2	15	3	36

6. Multiply 17 *lbs.* 5 *oz.* 12 *dwts.* 16 *grs.* Troy, by 9.

7. Multiply 4 *cwt.* 3 *qrs.* 8 *lbs.* by 28.

8. Multiply 17 *da.* 14 *hrs.* 14 *min.* 15 *sec.* by 12.

9. Multiply 16° 11′ 13″ by 7.

DIVISION OF COMPOUND NUMBERS.

223. Division of Compound Numbers is the operation of dividing a compound number by an abstract number, or by a similar denominate number.

The principles are the same as those used in the division of simple numbers.

Exercises for Oral Work.

1. Divide 35 nuts equally among 7 boys.

2. What is the quotient of 18*s.* divided by 6? By 9?

3. What is the quotient of 48 *yds.* divided by 12?

4. Divide 12*s.* 9*d.* by 3. Divide 18*s.* 6*d.* by 6.

5. What is the quotient of 84 *yds.* 3 *quarters* divided by 3?

6. Divide 57 *lbs.* by 19 *lbs.* Divide 114 *lbs.* by 19 *lbs.*

7. What is the quotient of 84 *yds.* divided by 12 *yds.*?

8. Divide 9 *yds.* 2 *ft.* 6 *in.* by 3. Divide 18 *yds.*

9. Divide 24° 30′ 36″ by 6. Divide 48° 24′ by 8.

10. A floor contains 40 *sq. yds.*, and its length is 8 *sq. yds.*; what is its breadth?

Operation of Division of Compound Numbers.

Let it be required to divide £65 18*s.* 8*d.* by 16.

ILLUSTRATION.

```
       £   s.  d.
16 )  65   18  8 ( £4  2s. 5d.
      64
      £1
      20
      38s.
      32
       6s.
      12
      80d.
      80
```

EXPLANATION.—Dividing £65 by 16 gives a quotient of £4 and a remainder £1. Reducing £1 to shillings and adding 18*s.*, we have 38*s.*; dividing 38*s.* by 16, gives a quotient 2*s.* and a remainder 6*s.* Reducing 6*s.* to pence and adding 8*d.*, we have 80*d.*; dividing 80*d.* by 16, we find a quotient 5*d* Hence the following

RULE.

I. Divide the units of the highest denomination in the dividend by the divisor; set down the quotient as a part of the required quotient.

II. Reduce the remainder to the next lower denomination, and to the result add the units of that denomination for a new dividend, and proceed as before.

III. Continue this operation till the division is completed.

EXAMPLES.

1. Divide 24 *da.* 19 *hrs.* 30 *min.* by 6.

2. Divide 43 *wks.* 2 *da.* 9 *hrs.* by 9.

3. Divide 39 *bu.* 2 *pks.* 1 *qt.* by 11.

4. Divide 1 *T.* 19 *cwt.* 2 *qrs.* 12 *lbs.* by 7.

5. Divide 56 *lbs.* 6 *oz.* 17 *dwts.* by 9.

6. Divids 14 *cwt.* 1 *qr.* 8 *lbs.* by 13.

7. Divide 13 *bu.* 3 *pks.* by 2 *bu.* 3 *pks.*

EXPLANATION.—The dividend is equal to 55 *pks.*, and the divisor is equal to 11 *pks.*; hence the quotient of 13 *bu.* 3 *pks.* ÷ 2 *bu.* 3 *pks.* is equal to 55 *pks.* ÷ 11 *pks.* = 5, *Ans.*

All similar cases may be worked by the following

RULE.

Reduce both numbers to the same denomination and divide as in simple numbers.

8. Divide 21 *wks.* 5 *da.* by 2 *wks.* 5 *da.*

9. Divide 2 *qrs.* 12 *lbs.* 9 *oz.* by 5 *lbs.* 11 *oz.*

10. Divide 19 *bu.* 3 *pks.* 3 *qts.* by 3 *bu.* 3 *pks.* 7 *qts.*

REVIEW QUESTIONS.

What is a bill? An account? A debtor? A creditor? How is a bill receipted? What is a footing? What is debit? Write the abbreviations of at, account, amount, balance, company, creditor, debtor, paid, and received. How do you find the footing of a bill? How the balance? What is reduction? Define reduction descending. Define reduction ascending. Recite the table of troy weight. What is its primary unit? Recite the table of avoirdupois weight. Recite the table of long measure. What is its unit of measure? What is magnitude? What is a line? A curved line? A straight line? What are parallel lines? Recite the table of surveyor's measure. For what is it used? What is a surface? An angle? Illustrate an angle, and name its parts. Define a square. A square foot. A square yard. What is the primary unit of the measure of surface? What is an area? How is the area of a square or rectangle found? Recite the table of square measure. Recite the table of fluid measure. For what used? What is its primary unit? Recite the table of dry measure. For what used? What is the primary unit? Describe the bushel. What is a circle? What the circumference? Diameter? Radius? Recite the table of angular measure. What is its primary unit? What is longitude? Recite the table of longitude. What is a volume? A cube? Faces of a cube? Edges? Recite the table of cubic measure. What is a cubic foot? A cord foot? A cord of wood? How do you reduce pounds sterling to shillings? How farthings to pounds? What fundamental rule is used in reduction descending? In reduction ascending? Tell the number of days in each month of the civil year. How can you tell when a year is a common year? How, when it is leap year? How many days in each? Is 1878 a leap year? Will the year 2000 be common, or leap year? How many days were there in the year 1800? When will the next leap year occur? What is compound addition? What compound numbers can be added? Give the rule for addition of compound numbers? What is subtraction of compound numbers? Give the rule. How do you find the interval between two dates? What is multiplication of compound numbers? Give the rule. What is division of compound numbers? How many cases are there? Give the rule when the divisor is abstract. Give the rule when the divisor is similar to the dividend.

NOTE.—For a more extended treatment of Denominate Numbers, see Davies & Peck's Complete Arithmetic.

PERCENTAGE AND ITS APPLICATIONS.

224. Per cent means **by the hundred, or hundredths.** Thus, 3 per cent of \$100 is $\frac{3}{100}$ of \$100, or \$3.

1. What is 2 per cent of 100? Of 300? Of 60?

2. How many dollars is 5 per cent of \$100? Of \$50?

3. How many *yds.* is 7 per cent of 100 *yds*? Of 500 *yds.*?

225. The sign of per cent is %. Thus, 3% of 20 is read 3 per cent of 20.

Read the following examples:

4. 4% of 20; 5% of 100; 6% of 30; 7% of \$200.

5. 1% of \$100; 9% of 27; 10% of 33 feet; 11% of 50.

6. 12% of 17 bushels; 20% of 450 ships; 30% of 72.

226. The rate per cent, or simply **rate,** is the number of hundredths taken; thus, in the expression 7% of 245, the rate is 7 *hundredths.*

1. How many per cent is .04? .06? .08? .07? .12? .17?

2. How many hundredths is 6%? 3%? 5%? 7%? 11%?

3. How many hundredths is 18%? 17%? 21%? 25%?

4. How many per cent is .25? .31? .15? .18? .09?

5. What is the decimal expression for $1\frac{1}{2}$%?

SOLUTION. $.01\frac{1}{2} = .015$.

Per cent expressed by means of a common fraction may be expressed decimally by annexing two ciphers to the denominator, and reducing the result to a decimal.

ILLUSTRATION. $\frac{4}{5}\% = \frac{4}{500} = .008$.

6. What decimal fraction is $\frac{1}{2}$%?

7. Express $\frac{3}{4}$% decimally. *8.* Express $\frac{2}{5}$% decimally.

9. Express $\frac{1}{6}$% decimally. *10.* What decimal fraction is $\frac{3}{5}$%?

11. How many hundredths is 100%? 200%?

12. How many hundredths is 125%? Express it decimally.

SOLUTION. 125% = $\frac{125}{100}$ = 1.25, *Ans.*

13. Express 250% decimally.

227. Percentage is some per cent of a given number. Thus, $6 is the percentage on $100, when the rate is 6 per cent.

1. What is the percentage on $200 at 5 per cent?

2. What is the percentage on $3528 at 2 per cent?

3. What is the percentage on $230 at 7 per cent?

4. What is the percentage on 350 yards, at rate of 3%?

5. Find the percentage on $15 at 50 per cent.

228. Base is the number on which percentage is reckoned. In the expression 7% of $100, the base is $100.

229. Amount is the base increased by the percentage. The amount of $100 at 8 per cent, is $100 + $8 = $108.

1. What is the amount of $200 at 3 per cent?

2. If 500 is the base, and 6 the rate, what is the amount? What, if 600 is the base, and 7 the rate?

3. If $648 is the base, and $16 the percentage, what is the amount?

4. Find the amount when the base is $4000, at 2%.

5. Find the amount when the rate is 6, and the base is $456.

230. The Difference is the base diminished by the percentage. Thus, the difference of $100 diminished by 8 per cent, is $100 — $8 = $92.

1. What is $600 diminished by 3 per cent?

2. Find the difference when the base is $875 and the percentage $26.25.

3. Find the difference when the base is $10,000 and the rate is 6%.

4. What is the difference when the base is $275 and the rate is 4%?

5. What is the difference when the base is $72 and the rate is 30%?

EXERCISES FOR ORAL WORK.

1. What is 5 per cent of 40 *lbs.*?

2. If $80 is increased by 5% of itself, what is the amount?

3. What is the difference of 80 yards diminished by 3 per cent of itself?

4. What per cent of $100 is $7? 8? 18? 20?

5. The base is $200 and the rate 7%; what is the amount? What is the difference?

6. A man had 30 chickens, 20 per cent of them were destroyed by foxes; how many were destroyed? How many were left?

7. How many marbles are 7% of 500 marbles?

8. A boy answered 25 questions in arithmetic, his brother answered 60% of that number; how many questions did the brother answer?

9. A man who held $10,000 worth of United States bonds, sold 10% of them; what value of bonds did he sell? What value of bonds had he left?

10. Let the base be $20,000 and the rate 6%; what is the percentage? What the amount? What the difference?

PRINCIPLES.

231. From what precedes we have the following principles:

1. *The percentage is equal to the base multiplied by the rate expressed decimally.*

2. *The amount is equal to the base multiplied by 1 plus the rate expressed decimally.*

3. *The difference is equal to the base multiplied by 1 minus the rate expressed decimally.*

Since either of two factors is equal to their product divided by the other, we have the following principles:

4. *The rate is equal to the percentage, divided by the base.*

5. *The base is equal to the percentage divided by the rate expressed decimally; to the amount divided by 1 plus the rate; or to the difference divided by 1 minus the rate expressed decimally.*

232. *To find the Percentage, when the base and rate are given.*

See Principle 1, Art. 231.

EXAMPLES.

1. What is 5% of 75 *lbs.*?

SOLUTION. 75 *lbs.* × .05 = *Ans.* 3.75 *lbs.*

2. What is 7% of 115 *lbs.*? Of 25 *lbs.*? Of 50 *lbs.*?

3. What is 11% of $315? Of $248? Of $600?

4. What is 16% of 52 *wks.*? Of 20 *wks.*? Of 50 *wks.*?

5. What is 25% of 4,120 *yds.*? Of 5640 *yds.*?

6. What is 40% of 72 *bu.*? Of 300 *bu.*?

7. What is 120% of $250? Of $840? Of $784?

8. What is 100% of 87 *ft.*? Of 500 *ft.*? Of 600 *ft.*?

9. What is 210% of 40 *gals.*? Of 63 *gals.*? Of 94 *gals.*?

10. What is 60% of $60? Of $500?

11. What is $\frac{3}{8}$% of $1,000? Of $8,000? Of $6,000?

233. *To find the Amount, when the Base and Rate are given.*

See Principle 2, Art. 231.

EXAMPLES.

1. What is the amount of 150 *lbs.* increased by 10% of itself?

150 *lbs.* × 1.10 = *Ans.* 165 *lbs.*

2. What is the amount of $300 increased by 35%?

3. What is the amount of 610 *yds.* increased by 16%?

4. What is the amount of 76 acres increased by 15%?

5. What is the amount of $218 increased by 9%?

6. What is the amount of 48 tons increased by 80%?

234. *To find the Difference, when the Base and Rate are given.*

See Principle 3, Art. 231.

EXAMPLES.

1. What is the difference between $108 and 30% of itself?

$108 × .70 = $75.60, *Ans.*

2. What is the difference between 160 *rods* and 80% of itself?

3. Diminish $540 by 30% of itself.

4. Diminish 64 weeks by 8% of itself.

5. Diminish 880 *yds.* by 30% of itself.

6. Diminish $1,050 by 7% of itself.

235. *To find the Rate, when the Base and Percentage are given.*

See Principle 4, Art. 231.

EXAMPLES.

1. The percentage is $90.24, and the base is $752; what is the rate?

$90.24 ÷ 752 = .12 = 12 per cent, *Ans.*

2. What is the rate, when the percentage is $7 and the base is $100? What, when the base is $500?

3. Find the rate, when the base is $400 and the percentage is $20.

4. What is the rate, when the percentage is 60 *lbs.* and the base is 300 *lbs.*?

5. Given the base $2,000, and the percentage $200; what is the rate?

6. A gentleman in Cleveland sends to his friend in New York $50,000, asking him to take out $500 for his services, and invest the balance in New York Central R. R. stock; what rate per cent of the money sent does he pay for the services of his friend? What rate per cent of the money invested does he pay?

236. *To find the Base, when the Rate and Percentage are given.*

See Principle 5, Art. 231.

EXAMPLES.

1. 960 is 25 per cent of what number?

SOLUTION. 960 ÷ .25 = *Ans.* 3,840.

2. 74 is 62½ per cent of what number?

3. 450 is 112 per cent of what number?

4. Of what number is 66, 2 per cent?

5. In a school 100 pupils are present; this number is 80 per cent of the pupils on the roll; what is the number on the roll?

MISCELLANEOUS PROBLEMS.

1. The population of a town in 1860 was 3,750, and in 10 years it increased 30%; what was the population in 1870?

2. A merchant bought goods to the amount of $3,150 and paid 15% on their first cost for transportation and insurance; what was their final cost?

3. A man has 50 geese, 40% more of chickens than geese, and 60% more of ducks than chickens; how many chickens has he, and how many ducks?

4. A cask of wine contained 44 *gals.*, but 18% of it leaked out; how much remained?

5. A man has a capital of $20,000, of which he loses 35%; how much has he remaining?

Define per cent. What is percentage? What is the base? What is the rate? Illustrate. What is the amount? What the difference? Repeat the 5 principles of percentage. Write an example that will illustrate each principle.

COMMISSION.

237. Commission is a percentage paid to an agent for transacting business.

238. An **Agent** is one who transacts business for another. If he buys and sells merchandise, he is called a **Commission Merchant,** or **Factor**; if he buys and

sells stocks, exchange, real estate and the like, he is called a **Broker**; if he collects debts, taxes, and the like, he is called a **Collector**.

239. A **Consignment** is a quantity of merchandise sent to an agent for sale. The party that sends the goods is called the **Consignor**; and the party to whom they are sent is called the **Consignee**.

240. An **Account of Sales** is an account rendered by the Consignee to the Consignor.

241. The **Net Proceeds** is the remainder after deducting commission and other expenses.

All problems in Commission are solved by the principles of percentage.

The base in Commission is generally what the agent expends or collects; but in buying and selling stocks, and the like, the commission is based on the par value.

EXAMPLES.

1. A commission merchant received a consignment of wheat which he sold for \$2,850; what was his commission at the rate of 4% ? *Ans.* \$114.

2. A commission merchant purchased cotton for a manufacturer to the amount of \$5,140; what was his commission at 2% ? *Ans.* \$102.80.

3. A broker sells a house for \$13,400; what is his commission at $1\frac{1}{8}$% ? *Ans.* \$150.75.

4. A broker buys 6 lots of land for \$8,490; what is his commission at $\frac{3}{8}$% ?

5. A cotton broker sells 40 bales of cotton, each weighing 450 *lbs.*, for 15 *cts.* per pound; what is his commission at $1\frac{3}{4}$% ?

6. My agent purchased goods for me to the amount of \$7,850, on which I agreed to pay $2\frac{1}{4}$% commission; how much must I pay him ?

7. An architect builds a house which costs \$13,710, and charges $2\frac{1}{2}$% for his services; what is his bill ?

8. A broker sold 70 shares of stock whose par value was \$100 per share; what was his commission at $\frac{1}{4}$% ?

9. What commission should a broker receive for selling 100 shares of stock whose par value is \$100, at $\frac{1}{4}$% ?

10. Find the net proceeds of the following account of sales:

Sales on account of JOHN R. BENSON, Rochester, N. Y.

1878.	DESCRIPTION.	\$	CTS.
Jan. 4	200 *bbls.* flour @ \$8 - - -		
17	115 *bu.* wheat @ \$1.10 -		
20	1000 *lbs.* butter @ 30*c.* - -		
	Gross amount		
	CHARGES.		
	Freight and cartage, \$48.75		
	Storage - - - - 10.50		
	Commiss'n on \$ @ 4% ____		
	Net proceeds - - - \$		

What is commission? What is an agent? A commission merchant? A broker? A collector? A consignment? A consignor? A consignee? What is the net proceeds? How do you solve problems in commission? What is the base? What is account of sales?

NOTE.—For a more extended treatment of Commission, see Davies & Peck's Complete Arithmetic.

PROFIT AND LOSS.

242. Profit and Loss are commercial terms indicating *gain* or *loss* in business transactions.

If the selling price of any article is greater than the cost price, there is a **profit**; if the selling price is less than the cost price, there is a **loss**.

The cost price is usually taken as the base.

Problems in profit and loss are solved by the principles of percentage.

EXAMPLES.

1. A man bought a horse for $220 and sold him at an advance of 15%; what did he gain?

The gain is the percentage.

The base is $220; hence, $220 × .15 = $33, the gain.

2. A merchant bought goods to the amount of $1,200, and sold them at a loss of 8%; how much did he lose?

3. A grocer bought sugar for $650, and sold it at an advance of 20%; what did he receive for it?

4. A dealer bought tea for $940, but was obliged to sell it at a loss of 12%; what did he get for it?

5. For what must a house, that cost $11,000, be sold so as to gain 15% on the cost price?

6. Coffee was bought at 22 *cts.* a pound; for what must it be sold to gain 20%?

7. A merchant bought 100 pieces of muslin at $7 a piece, and sold it at a gain of 16½%; what did he receive for the lot?

What is profit and loss? When is there a profit in business? When is there a loss in business? What is the base? By what principles do we solve problems in profit and loss?

NOTE.—For the application of percentage to Insurance. Taxes, etc., see Davies & Peck's Complete Arithmetic.

SIMPLE INTEREST.

243. Interest is a percentage paid for the use of money. It is reckoned at a certain rate per cent for each year.

The **Principal** is the sum on which interest is computed; the **Rate** is the per cent for 1 year; and the **Amount** is the sum of the principal and interest for any given time.

244. Legal Rate is the rate of interest fixed by law.

Interest depends on the *principal,* the *rate,* and the *time.*

EXERCISES FOR ORAL WORK.

1. What is the interest on $25 for 1 year, at 6 per cent?

$25 × .06 = *Ans.* $1.50.

2. What is the interest on $15 for 1 year, at 7%?

What is the *principal* in each of these examples?

What is the rate? What the time?

3. What is the interest on $30 at 7%, for two years?

$30 × .07 = $2.10 = int. for 1 year; the int. for 2 years is twice the int. for 1 year; $2.10 × 2 = $4.20, *Ans.*

4. What is the interest on $100 for 4 years, at 7%?

5. What is the interest on $200 for 3 years, at 6%?

6. What is the interest on $500 for 5 years, at 7%?

7. What is the amount of $300 for 1 year, at 7%?

The amount is the sum of the principal and interest?

$300 × .07 = $21, the int.; $21 + $300 = *Am't.* $321.

8. Find the amount of $200 for 2 years, at $6%.

9. What is the amount of \$300 for 3 years, at 7%?
10. Find the amount of \$400 for 2 years, at 7%?

245. ***When the time is given in years, the interest may be found by the following***

RULE.

Multiply the principal by the rate, and multiply that result by the number of years.

EXAMPLES.

1. What is the interest on \$85 for 4 years, at 7%?
Ans. \$85 × .07 × 4 = \$23.80.
2. What is the interest on \$96 for 3 years, at 6%?
Ans. \$96 × .06 × 3 = \$17.28.
3. What is the amount of \$336 for 5 years, at 5%?
4. What is the amount of \$242 for 3 years, at 7%?
5. What is the interest on \$425 for 5 years, at 7%?
6. What is the interest on \$650 for 2 years, at 7%?
7. What is the interest on \$780 for 3 years, at 6%?
8. What is the interest on \$850 for 5 years, at 6%?
9. What is the interest on \$1,200 for 7 years, at 7%?
10. What is the interest on \$1,400 for 3 years, at 7%?
11. What is the interest on \$890 for 4 years, at 5%?
12. What is the amount of \$510 for 3 years, at 7%?
13. What is the amount of \$1,240 for 4 years, at 7%?
14. What is the amount of \$1,780 for 3 years, at 6%?
15. What is the amount of \$1,672.15 for 4 years, at 7%?
16. What is the amount of \$1,587.73 for 3 years, at 6%?

246. ***To find the interest when the time is any number of months.***

ALIQUOT PARTS OF A YEAR.

12 mo. are 1 year,	6 mo. are $\frac{1}{2}$ of 1 year,
11 mo. are $\frac{11}{12}$ of 1 year.	5 mo. are $\frac{5}{12}$ of 1 year,
10 mo. are $\frac{5}{6}$ of 1 year,	4 mo. are $\frac{1}{3}$ of 1 year,
9 mo. are $\frac{3}{4}$ of 1 year,	3 mo. are $\frac{1}{4}$ of 1 year,
8 mo. are $\frac{2}{3}$ of 1 year,	2 mo. are $\frac{1}{6}$ of 1 year,
7 mo. are $\frac{7}{12}$ of 1 year,	1 mo. is $\frac{1}{12}$ of 1 year.

EXERCISES FOR ORAL WORK.

1. What is the interest on \$100 for 6 months, at 7%?

Since the interest on \$100 at 7% for 1 year is \$7, for 6 months, the half of 1 year, it is $\frac{1}{2}$ of \$7, or \$3.50. *Ans.*

2. What is the interest on \$200 for 3 months, at 7%?

Since the interest on \$200 at 7% for 1 year is \$14, for 3 months, the $\frac{1}{4}$ of 1 year, it is $\frac{1}{4}$ of \$14, or \$3.50. *Ans.*

RULE.

I. Find the interest for one year; take such part of this interest as the given number of months is part of 12 months or 1 year; or,

II. Multiply the principal by the rate and divide the product by 12; then multiply the quotient by the number of months.

EXAMPLES FOR WRITTEN WORK.

3. What is the interest on \$300 for 6 months, at 6%?
4. What is the interest on \$400 for 4 months, at 7%?

5. What is the interest on $200 for 5 months, at 6%?
6. Find the interest on $240 for 4 months, at 8%.
7. Find the interest on $553 for 5 months, at 6%.
8. Find the amount of $150.25 for 6 months, at 8%.
9. What is the amount of $204 for 11 months, at 7%?
10. What is the interest on $228 for 9 months, at 6%?
11. Find the amount of $137.50 for 8 months, at 6%.
12. What is the amount of $7596 for 10 months, at 8%?

RULE WHEN THE RATE IS 6 PER CENT.

Multiply the principal by half the number of months, and move the decimal point two places to the left hand.

247. ***To find the interest when the time is any number of days.***

For the sake of convenience in business transactions, 30 days are generally considered 1 month.

Aliquot Parts of a Month.

To find the aliquot parts of a month write the number of days for a numerator, and 30 for a denominator; and reduce the fraction to its lowest terms; Thus, 25 days are $\frac{25}{30} = \frac{5}{6}$ of a month; 15 days are $\frac{15}{30} = \frac{1}{2}$ of a month.

EXERCISES.

1. If the interest on a certain sum be $60 for 1 month, how many dollars will it be for 1 day?

Since 1 day is $\frac{1}{30}$ of 1 month, the interest for 1 day is $\frac{1}{30}$ of the interest for 1 month; $\frac{1}{30}$ of $60 = $2, *Ans.*

2. The interest on $400 for 1 month, at 9%, is $3; what is the interest on $400, at 9%, for 15 days?

Since 15 days are the $\frac{1}{2}$ of 1 month, the interest for 15 days is $\frac{1}{2}$ of the interest for 1 month; $\frac{1}{2}$ of $3 is $1.50, *Ans.*

3. If $6 is the interest of a certain sum of money for 1 month, what is the interest of the same sum for 20 days?

Since 20 days are $\frac{2}{3}$ of 1 month, the interest for 20 days is $\frac{2}{3}$ of $6, or $4, *Ans.*

4. The interest on $900 at a certain rate for 1 month is $5.25; what is it for 6 days?

Since 6 days are $\frac{1}{5}$ of 1 month, the interest for 6 days is $\frac{1}{5}$ of $5.25 = $1.05, *Ans.*

From these illustrations we deduce the following

RULE.

I. Find the interest for 1 month, take such a part of this interest as the given number of days is part of 30 days or 1 month; or,

II. Divide 1 month's interest by 30, and multiply the quotient by the given number of days.

EXAMPLES.

1. What is the interest on $460 for 15 days, at 7%?
2. What is the interest on $784, at 7%, for 20 days?
3. What is the interest on $1200 for 10 days, at 6%?
4. What is the interest on $184 for 21 days, at 7%?
5. What is the interest on $250 for 18 days, at 6%?
6. What is the interest on $375 for 25 days, at 7%?
7. What is the interest on $450 for 6 days, at 9%?
8. What is the interest on $500 for 7 days, at 7%?

248. ***To find the interest when the time is in years, months, and days.***

Find the interest on $360 for 3 years 7 months and 25 days.

ILLUSTRATION.

```
          360
          .07
  2 )   25.20   Int. for 1 yr.
            3
        75.60   Int. for 3 yrs.
    6 ) 12.60   Int. for 6 mo.  }
3 ) 2 )  2.10   Int. for 1 mo.  } 7 mo.
         1.05   Int. for 15 da. }
           70   Int. for 10 da. } 25 da.
       $92.05   Int. for 3 yrs. 7 mo. 25 da.
```

EXPLANATION.—We find the int. for 1 year by multiplying the *principal* by the rate expressed decimally. We multiply the int. of 1 year by 3, to find the int. for 3 years. We then take ½ of the int. of 1 year for the int. for 6 *mo.*, and ⅙ of the interest for 6 months for the interest for 1 month. We then take ½ of the interest for 1 month for the interest for 15 days, and ⅓ of the interest for 1 month for the interest for 10 days; hence the interest for 3 *yrs.* 7 *mo.* 25 *da.* is $92.05.

SECOND ILLUSTRATION.

```
12 ) 25.20                 $360
      2.10                  .07
         7                25.20   Int. for 1 year.
    $14.70                    3
                          75.60   Int. for 3 years.
30 ) 2.10                 14.70   Int. for 7 mos.
      .07                  1.75   Int. for 25 days.
       25                $92.05   Int. for 3 years,
       35                           7 months, and
      14                            25 days.
     1.75
```

EXPLANATION.—We find the int for 1 *yr.* and for 3 *yrs.*, as in the first illustration. We then divide 1 year's int. by 12, which gives us $2.10, the int. for 1 *mo.* Multiply the int. for 1 *mo.* by 7, obtaining $14.70, the int. for 7 mo. We then divide the int. for 1 *mo.* by 30 to find the int. for 1 *da.*, and multiply this int. by 25 to find the int. for 25 *da.*, which is $1.75. We then place the int. for 7 *mo.* and the int. for 25 *da.* under the int. for 3 years, and find their sum, $92.05.

RULE.

Find the interest for each period and add the results.

EXAMPLES FOR WRITTEN WORK.

1. What is the interest on $174 for 3 *yrs.* 2 *mos.*, at 6%? *Ans.* $33.06.

2. What is the interest on $350 for 2 *yrs.* 6 *mos.*, at 7%? *Ans.* $61.25.

3. What is the interest on $700 for 4 *yrs.* 7 *mos.*, at 7%?

4. What is the interest on $825 for 2 *yrs.* 9 *mos.*, at 6%?

5. What is the amount of $950 for 3 *yrs.* 6 *mos.*, at 7%?

6. What is the amount of $686 for 3 *yrs.* 8 *mos.*, at 6%?

7. What is the amt. of $1,218 for 1 *yr.* 11 *mos.*, at 7%?

8. What is the amt. of $1,250 for 2 *yrs.* 3 *mos.*, at 6%?

NOTES.

249. A **Promissory Note** is a written promise to pay a sum of money, either on demand, or at some particular time.

The party that signs the note is called the **Maker,** the party to whom it is payable is called the **Payee,** and the party that has legal possession of it is called the **Holder.**

A **Negotiable Note** is one that is payable either *to order*, or *to bearer;* the following is the

FORM OF A NEGOTIABLE NOTE.

$375.00. New York, July 2d, 1878.

For value received I promise to pay John Doe, or order, three hundred and seventy-five dollars on demand, with interest at 7 per cent.

Richard Roe.

John Doe is the *payee*. He can transfer it by writing his name across the back; he then becomes the *endorser*. Richard Roe is the *maker*, and the person to whom it is *transferred* becomes the *holder*.

The **Face** of the note is the sum named in it.

250. To find the interest on a note we have the following

RULE.

I. Subtract the date of the note from the date of settlement; the result will be the time.

II. Find the interest on the face of the note for this time.

EXAMPLES.

1. A note for $250 at 7%, bears date Oct. 10, 1877; how much interest will be due Dec. 13, 1878.

ILLUSTRATION.

yr.	*mo.*	*da.*
1878	12	13
1877	10	10
1 *yr.*	2 *mo.*	3 *da.*

EXPLANATION.—Here the time is 1 *yr.* 2 *mos.* 3 *da.*, and the interest on $250 for this time at the rate of 7%, is $20.562, *Ans.*

2. A note for $780 at 7%, bears date July 5, 1870; how much interest is due May 17, 1873? *Ans.* $156.52.

3. A note for $960 at 6% is dated Dec. 24, 1871; what amount is due on the note June 12, 1874?

4. A note for $1,140 at 7% is dated Jan. 11, 1872; what is its amount June 5, 1874?

PARTIAL PAYMENTS.

251. A **Partial Payment** is a payment of a *part* of the amount due on a note or other obligation.

The date and amount of each partial payment is *indorsed*, that is, *written on the back of the note*, and is to be taken into account at the settlement.

The following rule, for settling a note with endorsements, is adopted by the Supreme Court of the United States, and is called

THE SUPREME COURT RULE.

I. Find the amount of the given principal up to the time when the sum of the partial payments equals, or exceeds, the interest then due; from this subtract the sum of the partial payments up to the time considered.

II. Take the remainder for a new principal and proceed as before, continuing the operation to the time of settlement.

EXAMPLES.

1. On a note for $1,250 at 6%, dated June 10, 1876, are the following endorsements:

December 16, 1876, $30.
July 4, 1877, $250.

What is the amount due August 25, 1878?

OPERATION.

Principal, . . .	$1250
Int. to July 4, 1877, .	80
Am't July 4, 1877, .	$1330
Sum of payments, .	280
Remainder, ...	$1050
Int. to Aug. 25, 1878,	71.925
Amount, . .	$1121.925

EXPLANATION.—We see by inspection that the first payment is less than the interest then due; we therefore find the amount of the note up to the date of the second payment, which is $1,330; from this we subtract the sum of the first and second payments, and then find the amount of the remainder from the time of the second payment up to the date of settlement; this gives $1,121.925.

2. On a note for $960 at 7%, dated March 17, 1875, are the following endorsements:

March 17, 1876, $250.
March 17, 1877, $350.

What is due March 17, 1878? *Ans.* $515.3163.

DISCOUNT

252. Discount is a percentage deducted from the face of a bill, debt, or note.

COMMERCIAL DISCOUNT.

253. Commercial Discount is a percentage deducted from the face of a bill of merchandise.

The face of the bill is the **Base**, and the difference between this and the discount is called the **Net Proceeds.**

To find commercial discount we have the following

RULE.

I. Multiply the face of the bill by the rate per cent, and the product will be the discount.

II. Subtract the discount from the face of the bill, and the difference will be the net proceeds.

1. What is the discount on a bill of $350 at 2%? What the net proceeds? *Ans.* $7; $343.

2. Coal is sold on credit at $5 per ton; what is the cost price, the discount being 12%? *Ans.* $4.40.

3. Bought a bill of goods whose face is $1,300, at a discount of $2\frac{1}{2}$%; how much must I pay? *Ans.* $1267.50.

4. Sold 50 *bbls.* of flour at $7.50 per *bbl.*, deducting $7\frac{1}{2}$% for cash; what was the net proceeds? *Ans.* $346.875.

PRESENT VALUE AND TRUE DISCOUNT.

254. The **Present Value** of a debt, payable at a future time, is a sum which, being placed at interest, will give an amount equal to the debt when it falls due.

255. True Discount is the difference between the amount of the debt and its present value. Hence the

RULE.

I. Divide the amount of the debt by $1 plus the interest of $1 for the given time and at the given rate; the quotient will be the present value.

II. Subtract the present value from the amount of the debt; the remainder will be the true discount.

1. What is the present value of $500 due 2 years hence, money being worth 7%?

2. What is the true discount on a debt of $600 due 1 year hence, if paid now, at 6%?

BANKS AND BANK DISCOUNT.

256. **A Bank** is an incorporated institution, authorized by law to deal in money.

257. **Bank Discount** is a percentage charged for advancing money on a note or other obligation, payable at a future time.

258. A note is said to **Mature** when it becomes legally due, which is 3 days after it is nominally due. These 3 days are called **Days of Grace.**

Form of Bank Note.

$1000. New York, March 2, 1878.

Sixty days after date I promise to pay to the order of Alfred C. Barnes, one thousand dollars, at the Metropolitan Bank, New York, for value received.

Charles Martin.

Due May 1/4, 1878.

Explanation.—Suppose this to be discounted on the day of its date. Alfred C. Barnes endorses it by writing his name across the back, and delivers it to the discount clerk. Interest is then computed on the face of the note for 63 days—the days of grace being added to the time mentioned in the note—at 7%, the legal rate in New York. This sum, $12.08, is the Bank discount. Subtracting the discount from $1000 we have $987.92 the proceeds. If the note is not paid before the close of banking hours of the last day of grace, May 4, a written notice, called a ***Protest***, is sent to Mr. Barnes, and he then becomes liable for its payment.

EXAMPLES.

1. A note for $2000, payable 60 days after date, is discounted at the rate of 7%; what is the proceeds?

2. A note for $3000, dated April 5, 1878, and payable 90 days after date, at 7%, is discounted on the day of its date; what is the proceeds?

REVIEW QUESTIONS.

What is interest? What is the principal? The rate? The amount? On what three elements does interest depend? Give the rule for interest when the time is given in years. When the time is given in months. Give the rule for interest when the time is given in days. Give the rule for interest when the time is given in years, months, and days. What is a promissory note? Who is the maker? The payee? The holder? When does the payee become an endorser? What is the rule for finding the interest on a note? What is a partial payment? How endorsed? Give the Supreme Court rule for settling a note with endorsements. What is discount? What is commercial discount? Give the rule to find commercial discount. What is present value? What is true discount? Give the rule to find true discount. What is a Bank? What is Bank discount? When is a note said to mature? What are days of grace? Write the form of a Bank note. Explain the method of discounting a Bank note.

Note.—For a full treatment of percentage and its applications, see Davies & Peck's Complete Arithmetic. See the same work for a full treatment of Proportion, Analysis, Square and Cube Roots, Mensuration, etc.

MISCELLANEOUS EXAMPLES.

1. A wagon wheel that turns round 346 times in running 1 mile, turns round 32,870 times in running from New York to Philadelphia: what is the distance between the two cities? *Ans.* 95 miles.

2. A gentleman bought 4 loads of hay; the first load weighed $1\frac{1}{4}$ tons, the second weighed 1 ton, the third weighed $\frac{1}{2}$ as much as the first and second together, and the fourth weighed $1\frac{1}{2}$ tons: how much hay did he buy? *Ans.* $4\frac{7}{8}$ tons.

3. If \$54 dollars will purchase 9 barrels of flour, how many barrels will \$186 buy? *Ans.* 31 barrels.

4. A ship is 97 *ft.* 6 *in.* long; how many times her own length does she sail in running 78 miles? *Ans.* 4,224 times.

5. What is the bank discount on a note for \$400, payable 90 days after date, at 6%? *Ans.* \$6.20.

6. A. owned $\frac{1}{3}$ of a ship, and sold $\frac{4}{5}$ of his share for \$13,500; what would the entire ship bring at the same rate? *Ans.* \$50,625.

7. A. can do $\frac{2}{3}$ as much work as B., and B. can do $\frac{4}{5}$ as much as C.; if C. can do a piece of work in 10 days, how long will it take A. to do it? *Ans.* $18\frac{3}{4}$ days.

8. The longitude of Nantucket is 70° 5′ 39″, and the longitude of New York City is 74° 0′ 3″; what is the difference of longitude of the two places? *Ans.* 3° 54′ 24″.

9. A commission merchant received a consignment of corn, which he sold for \$894, receiving a commission of $3\frac{1}{2}$%; what was his commission? *Ans.* \$31.29.

10. A. and B. commenced business on the 1st of January with a capital of \$5,000 each; during the year A. lost 40% of his capital, and B. gained 30% of his capital; how much more capital had B., at the end of the year, than A.? *Ans.* \$3,500.

11. A. borrows \$580 at a bank, payable in 60 days; what is the net proceeds, interest being reckoned at 7%? *Ans.* \$572.895.

12. What is the cost of \$4,000 of U. S. 5-20's at 110%, and brokerage at $\frac{1}{4}$%? *Ans.* \$4,410.

13. If $7\frac{3}{16}$ barrels cost \$$24\frac{1}{4}$, what will 1 barrel cost? *Ans.* $3\frac{43}{115}$.

14. A passenger train leaves New York for Albany, distant 144 miles, at 8 o'clock in the morning, and travels at the rate of 23 miles an hour; at 10 o'clock a freight train leaves Albany for New York, and travels at the rate of 11 miles an hour: how far are the trains apart at 12 o'clock? *Ans.* 30 miles.

15. Four men bought a tract of land for \$10,463; the first paid \$2,814, the second paid \$987 less than the first, the third paid \$149 more than the first and second together, and the fourth paid the rest: how much did the fourth pay? *Ans.* \$1,032.

16. A man sold 21 barrels of apples at \$4 a barrel, and 18 bushels of wheat at \$2 a bushel, for which he received \$64 in cash, and the rest in flour at \$14 a barrel; how much flour did he receive? *Ans.* 4 barrels.

17. A man exchanges 380 gallons of molasses, at \$$\frac{3}{4}$ per gallon, for $33\frac{1}{4}$ hundredweight of cheese, at \$8 per hundredweight; what is the balance in his favor? *Ans.* \$19.

18. A man owned $\frac{3}{8}$ of a farm, and sold $\frac{5}{6}$ of his share for \$2,345; what was the value of the whole farm at the same rate? *Ans.* \$7,504.

19. A farmer sold 70 bushels of rye at \$$1\frac{1}{8}$ per bushel, and received for the same \$47 in cash, and the balance in muslin at \$$\frac{3}{16}$ per yard; how many yards of muslin did he receive? *Ans.* $169\frac{1}{3}$ yards.

20. A market-woman bought 246 eggs at the rate of 2 eggs for 3 cents, and sold them again at $1\frac{3}{4}$ cents apiece; how much did she make by the purchase? *Ans.* $61\frac{1}{2}$ cents.

21. A grocer bought 126 gallons of molasses at \$.75 per gallon; he lost 18 gallons by leakage, and then sold the rest so as to clear \$27: at what price per gallon did he sell it? *Ans.* \$$1.12\frac{1}{2}$.

22. If a man can travel 4.3 miles in 1 hour, how long will it take him to travel 50.31 miles? *Ans.* 11.7 hours.

23. From a piece of cloth containing 34.7 yards, a tailor cut enough to make 3 suits of clothes, each suit requiring 5.3 yards; how many yards remained? *Ans.* 18.8 yards.

24. Bought 6 pieces of linen, each piece containing 34.3 yards, at $62\frac{1}{2}$ cents a yard, and sold the entire lot so as to make \$$25.87\frac{1}{2}$; what was the selling price per yard? *Ans.* \$$0.75\frac{25}{343}$.

25. Two men set out from the same place, and travel in the same direction; the first travels at the rate of 5.3 miles per hour, and the second at the rate of 3.9 miles per hour: how far apart are they at the end of 7.5 hours? *Ans.* $10\frac{1}{2}$ miles.

26. A man left an estate of \$41,520 to be distributed as follows: \$3,000 to a charitable institution, $\frac{1}{3}$ of the remainder to his wife, and the rest equally to his 4 children; what was the share of each child? *Ans.* \$6,420.

27. From a board 13 *ft.* 7 *in.* long, a carpenter sawed off a piece 8 *ft.* 11 *in.* long; how long was the remaining piece? *Ans.* 4 *ft.* 8 *in.*

28. The latitude of Baltimore is 39° 17′ 48″, and the latitude of Savannah is 32° 4′ 53″; what is the difference of latitude of the two places? *Ans.* 7° 12′ 55″.

29. The fore-wheels of a wagon are 12 *ft.* 6 *in.* in circumference, and the hind-wheels are 15 *ft.* in circumference; how many more times will the fore-wheels turn than the hind-wheels, in running a distance of 25 miles? *Ans.* 1,760 times.

30. A man bought 18 silver spoons, each weighing 3 *oz.* 5 *dwts.*; 24 teaspoons, each weighing 15 *dwts.* 14 *grs.*; 3 cups, each weighing 9 *oz.* 7 *dwts.*; 2 tankards, each weighing 21 *oz.* 15 *dwts.*; and 6 porringers, each weighing 11 *oz.* 18 *dwts.*: what was the weight of the whole? *Ans.* 18 *lbs.* 4 *oz.* 3 *dwts.*

31. An Englishman owned $\frac{2}{5}$ of a ship, and sold $\frac{5}{8}$ of his share for £375; what was the value of the entire ship in U. S. currency? *Ans.* \$7,299.75.

32. If the moon moves through 13° 10′ 35″ in 1 day, how long will it take her to move through 360°? *Ans.* 27 *da.* 7 *h.* 43 *m.* 6 *sec.*

33. The shadow of a vertical staff on level ground is 11 feet long, and the shadow of a tree at the same time is $82\frac{1}{2}$ feet long; if the height of the staff is $5\frac{1}{4}$ feet, what is the height of the tree? *Ans.* $39\frac{3}{8}$ feet.

34. A horse was worth \$175 in the fall, and during the winter he increased in value 17%; what was he worth in the spring? *Ans.* \$$204\frac{3}{4}$.

35. A dealer bought 36 *bbls.* of apples at \$4 per *bbl.*; 9 *bbls.* proving defective, he sold them at a loss of 40% on their cost, and the remainder he sold at an advance of 35% on their cost; how much did he make by the transaction? *Ans.* \$23.40.

36. A man distributed \$9,000 amongst his 3 sons as follows: to his oldest son he gave 40% of the whole, to his second son he gave 60% of the remainder, and to his third son he gave what was left; what did the third son receive? *Ans.* \$2,160.

37. A man bought 75 shares of Reading R. R. stock, par value \$50 per share, at the rate of $112\frac{3}{4}$%; what was the cost, including brokerage of $\frac{1}{4}$% on the par value? *Ans.* \$4,237.50.

38. When is the following note legally due, and what is its net proceeds?

\$780. New York, June 11, 1878.

Sixty days after date, for value received, I promise to pay to John Nettre, or order, seven hundred and eighty dollars with interest at 6%.

Henry Waltham.

Ans. Aug. 13th, 1878. Net proceeds, \$771.81.

39. What is the amount of \$580 for 1 year 6 months, at 7%? *Ans.* \$640.90.

40. A produce dealer sold 50 bushels of corn at \$1.12 per bushel, and 25 bushels of buckwheat at 90 cents a bushel; if he allowed a discount of 5% for cash, what was the proceeds? *Ans.* \$$74.57\frac{1}{2}$.

41. A speculator bought two lots of ground, each costing \$2,500; on the first he made 33%, and on the second he lost 17%; what was his entire gain? *Ans.* \$400.

42. A man went into business with a capital of \$4,500: the first year he made 18% on his capital, the second he lost 5% on the amount of capital he had at the end of the first: what did he gain in the two years? *Ans.* \$544.50.

43. A man sold 52 shares of N. Y. C. R. R. stock, par value $100 at $106\frac{1}{2}\%$, and paid brokerage at $\frac{1}{4}\%$; what was the net proceeds? *Ans.* $5,525.

44. A merchant bought a lot of goods for $8,200, and his expense for freight, transportation, commission, and interest was 8% on the prime cost; for what must he sell them to realize 15% on the total cost? *Ans.* $10,184.40.

45. An area 50 *ft.* long and 38 *ft.* 6 *in.* wide is to be paved with flag-stones, each 2 *ft.* 6 *in.* long, and 1 *ft.* 9 *in.* wide; how many stones will it require? *Ans.* 440 stones.

BOARD MEASURE.—A volume that is 1 foot long, 1 foot wide, and 1 inch thick, is called a *board foot.* It is equal to $\frac{1}{12}$ of a cubic foot. The contents of boards, planks, and some kinds of timber are frequently expressed in board feet. The number of board feet in such a volume may be found by the following

RULE.—*Multiply the breadth by the thickness, both in inches, and that result by the length in feet; then divide the product by 12.*

46. How many board feet of lumber will it take to build a platform 18 *ft.* 4 *in.* long, 12 *ft.* wide, and $3\frac{1}{2}$ *in.* thick? *Ans.* 770 board feet.

47. A pile of wood is 32 feet long, 4 feet wide, and $3\frac{1}{2}$ feet high; how many cords does it contain? *Ans.* $3\frac{1}{2}$ cords.

48. A pile of wood is 40 feet long and 4 feet wide; how high must it be to contain $6\frac{1}{4}$ cords? *Ans.* 5 feet.

49. How many cords of wood are there in a pile 15 feet long, 3 feet wide, and 6 feet high? *Ans.* 2 *C.* 14 *cu. ft.*

50. A pile of wood is 12 *ft.* 6 *in.* long, 3 *ft.* 9 *in.* wide, and 4 *ft.* 2 *in.* high; how many cords does it contain? *Ans.* 1 *C.* $67\frac{5}{16}$ *cu. ft.*

51. A man wishes to build a slat fence 480 feet long; for this purpose he requires 81 posts, each 6 inches square and 8 feet long, and 160 slats each 12 feet long, 7 inches wide, and 1 inch thick; how many board feet of lumber does he require? *Ans.* 3,064 board feet.

ANSWERS.

Answers are given to all the written and the more difficult of the oral examples. The references are to the page. When two articles with examples of like numbers are on the same page, the references are to both page and article. The number on the left of each column is the number of the example, that on the right is the answer.

P. 39.
1. 177.
2. 189.
3. 209.
4. 270.
5. 93.
6. 144.
7. 220.
8. 202.
9. 191.
10. 128.
11. 209.
12. 242.

P. 40.
13. 1,358.
14. 758 da.
15. 1,109 qrt.
16. 1,688.
17. 1,998.
18. 1,357 yrs.
19. 2,077 qts.
20. 21,953 lbs.
21. 1,633.
22. 3,498.
23. 3,331.
24. 2,638 da.

P. 41.
25. 3,075 ft.
26. 3,373 yds.
27. 1,496 lbs.
28. 3,760 in.
29. $3,675.
30. 371,778.
31. $74,719.
32. 129,479 yds.
33. 298,945 lbs.
34. 189,911 ft.
35. $144,230.
36. 102,749 in.

P. 42.
8. $1,795.
9. $137.
10. 370 mi.

P. 43.
WRITTEN.
1. $1015.
2. $8,280.
3. 3,911 bu.

P. 44.
4. 3,555.
5. 10,148.
6. 1,010 mi.
7. $86,312.
8. $2,800.

P. 51.
1. 53.
2. 41.
3. 32.
4. 45.
5. 31.
6. 26.
7. 32 yds.
8. 33 yds.
9. $11.
10. 42 lbs.
11. 72 in.
12. 52 ft.

P. 53.
1. 32.
2. 25.
3. 37.
4. 6.
5. 16.
6. 19.
7. 235.
8. 612.
9. 563.
10. 611.
11. 531.
12. 363 lbs.
13. 212 ft.
14. 445 yds.
15. $552.
16. 491 in.

P. 54.
17. 1,165.
18. $4,557.
19. 1,027.
20. 6,712 ft.
21. 2,774.
22. 1,832.
23. 86,222.
24. 27,087.
25. 4,014.
26. 173.
27. 631.
28. 7,832.
29. $7,675.
30. 96.081 ft.
31. 973,069.
32. $3,489.
33. 66,481 yds.
34. $462.
35. 10,525 yds.
36. 629,629.
37. 4,909 ft.
38. 136,783.
39. 3,045.

P. 55.
7. 282 yds.
8. 286 sheep.
9. $159.
10. $256.
11. $1,875.

P. 56.
7. $12,586.
8. 286 sheep.
9. $18,589.
10. $550.
11. 559 sheep.

P. 57.
6. 431 men.
7. $3,258.
8. $3,353.
9. $506.
10. $984.

P. 58.
7. Second, $5.
8. $418.
9. $5,795.
10. $390.

P. 59.
11. 1714.
12. 30,927.
13. 398.
14. 25,933.
15. 6,760.
16. $6,740.

P. 66.
2. 1,272.
3. 512.
4. 3,232.
5. 1,590.
6. 3,332.
7. 9,872.
8. 37,233.
9. 2,502 ft.
10. 12,901 in.
11. $4,470.
12. 6,237 lbs.
13. 25,160.
14. 302,498 lbs
15. 488,052.
16. $49,374.

P. 68.
3. 266,616.
4. 164,208.
5. 630,975.
6. 131,577.
7. 256,335.
8. 589,536.
9. 74,088.
10. 578,856.
11. 36,914,176.

12. 85,950,000.
13. 3,320,863,272.
14. 816,515,040.
15. 68,959,488.
16. 6,241,519,790.
17. 105,062,176.
18. 294,360,066.
19. 173,637.
20. 77,896.
21. 504,684.
22. 156,618.
23. 247,932.
24. 318,364.
25. 683,769.
26. 624,064.
27. 170,280.
28. 110,982.
29. 4,324.
30. 43,562.
31. 99,924.
32. 239,184.
33. 133,416.
34. 624,635.

P. 69.

1. 6360.
2. 822,600.
3. 4,071,500.
4. 334,000.
5. 262,200 yds.
6. $58,870.
7. 32,885,600.
8. 162,400.
9. 412,800.
10. 434,000.
11. 171,600.
12. $16,227.
13. 1,112,400.
14. 1,150,050.
15. 261,000.
16. 3,771,000.
17. 1,250,000.
18. 11,200,000.

P. 70.

19. 56,700,000.
20. 365,000.
21. 276,800.
22. $1,425,900.
23. 3,090,000 yds.
24. 1,410,000.
25. 120,000.
26. 355,000.
27. 2,347,200.
28. 126,000 ft.

P. 71.

21. 3,744.
22. 10,368.
23. 23,128.
24. 1,848.
25. 52,416.
26. 700,728.
27. 1,680.
28. 51,840.

P. 72, Art. 65.

7. $94.50.
8. $156.14.
9. $70.74.
10. 777 tickets.

P. 72, A. 66.

6. 328 lbs.
7. 136 lbs.
8. 86,400 sec.

P. 73.

10. $806.40.
11. $1,432.

P. 73, A. 67.

8. $266.40.

P. 74.

9. 1,104 mi.
10. $23.78.
11. 21,284.
12. 3,432,000.
13. 719.
14. 32,456,909.
15. 246,099.
16. 4,846,800 yds.

P. 82.

1. 157.
2. 103.
3. 223.
4. 342.

P. 83.

6. $64\frac{4}{5}$.
7. $1,587\frac{1}{3}$.
8. $110\frac{2}{9}$.
9. $535\frac{2}{3}$.
10. 2,704.
11. 3,049.
12. $3,229\frac{1}{5}$.
13. 1,281.
14. $12,648\frac{1}{5}$.
15. 283.
16. $386\frac{1}{5}$.
17. $10,292\frac{2}{11}$.

P. 85.

1. $34\frac{4}{25}$.
2. 312.
3. 38.
4. 54.
5. 64.
6. 192.
7. 312.
8. $404\frac{54}{55}$.
9. 371.
10. 108.
11. 223.
12. 215.
13. $58.
14. 594.
15. $564.
16. 259 lbs.
17. 118 lbs.
18. 79 yds.
19. 346.
20. 245.
21. $63\frac{128}{214}$.
22. $56\frac{4}{47}$.
23. $94\frac{1}{32}$.
24. $13\frac{5}{235}$.

P. 86.

1. $21\frac{334}{400}$.
2. $5\frac{4121}{6000}$.
3. $204\frac{781}{900}$.
4. $15\frac{64}{2500}$.
5. $17\frac{543}{16000}$.
6. $24\frac{452}{1500}$.

P. 87.

7. 5.
8. 30.
9. 163.
10. 250.
11. $3\frac{32}{160}$.
12. $22\frac{1470}{3000}$.

P. 88.

6. { 52 da., $17\frac{1}{3}$ da., 13 da.
7. $37, $74, $111.
8. 12 da., 24 da., 48 da.
9. 144 min., 72 min.
10. $283, $141.50.

P. 89.

6. 12 da.
7. $240, $3,120.
8. 238 w.
9. $70.
10. $780, $242, $478.

P. 90.

11. 40 lbs.
12. 63.
13. 2,684 lbs.

P. 91.

14. 97.
15. $4.
16. $108.
17. 46 lbs.
18. 351 A.
19. $1.40 gain.
20. 30 h.
21. 102 da.
22. 86 A., $42.

P. 97.

1. $2 \times 3 \times 5 \times 7$.
2. $2 \times 3 \times 5 \times 11$.
3. $5 \times 7 \times 29$.
4. $2 \times 2 \times 3 \times 13$.
5. $2 \times 5 \times 31$.
6. $3 \times 3 \times 5 \times 11$.
7. $5 \times 5 \times 17$.
8. $2 \times 3 \times 3 \times 5 \times 11$.
9. $3 \times 5 \times 5 \times 13$.
10. $3 \times 3 \times 5 \times 17$.
11. $2 \times 3 \times 5 \times 19$.
12. $2 \times 2 \times 2 \times 3 \times 3 \times 7$
13. $3 \times 3 \times 3 \times 5 \times 11$.
14. $3 \times 5 \times 5 \times 5 \times 7$.
15. $2 \times 3 \times 3 \times 3 \times 17$.
16. $2 \times 3 \times 3 \times 19$.
17. $2 \times 2 \times 2 \times 103$.
18. $2 \times 2 \times 2 \times 79$.
19. $2 \times 2 \times 137$.
20. $2 \times 5 \times 5 \times 7$.

P. 100.

5. 5.
6. 5.
7. $1\frac{14}{83}$.
8. $1\frac{73}{99}$.
9. $1\frac{5}{17}$.
10. $3\frac{1}{3}$.
11. $12\frac{5}{6}$.
12. $27\frac{30}{31}$.
13. $234\frac{2}{3}$.

P. 101.

1. 6.
2. 14.
3. 42.
4. 5.
5. 15.
6. 9.

P. 101.

1. 4.
2. 3.
3. 11.
4. 47.
5. 189.
6. 267.

P. 103.

1. 60.
2. 120.
3. 72.
4. 180.
5. 180.
6. 840.
7. 432.
8. 840.

P. 110.

1. $\frac{3}{4}$.
2. $\frac{2}{4}$.
3. $\frac{5}{7}$, $\frac{3}{5}$.
4. $\frac{4}{5}$.
5. $\frac{13}{8}$.

P. 111.

2. $\frac{2}{5}$.
3. $\frac{7}{27}$.
4. $\frac{12}{13}$.
5. $\frac{11}{15}$.
6. $\frac{23}{25}$.
7. $\frac{18}{25}$.
8. $\frac{13}{81}$.
9. $\frac{2}{3}$.
10. $\frac{55}{13}$.
11. $\frac{144}{125}$.
12. $\frac{11}{9}$.
13. $\frac{5}{13}$.
14. $\frac{314}{7}$.
15. $\frac{21}{1}$.
16. $\frac{12}{1}$.
17. $\frac{13}{14}$.
18. $\frac{21}{32}$.
19. $\frac{3}{5}$.
20. $\frac{13}{17}$.
21. $\frac{17}{19}$.
22. $\frac{9}{13}$.
23. $\frac{11}{15}$.
24. $\frac{10}{29}$.
25. $\frac{23}{25}$.
26. $\frac{56}{205}$.
27. $\frac{8}{13}$.
28. $\frac{25}{33}$.
29. $\frac{877}{969}$.
30. $\frac{9}{14}$.
31. $\frac{37}{61}$.

P. 112.

3. $4\frac{5}{9}$
4. $25\frac{11}{12}$.
5. $43\frac{7}{11}$.
6. $25\frac{1}{13}$.
7. $14\frac{13}{15}$.
8. $63\frac{1}{7}$.
9. $57\frac{11}{12}$.
10. $3\frac{9}{22}$.
11. $7\frac{13}{14}$.
12. $27\frac{5}{11}$.
13. $106\frac{1}{8}$.
14. $100\frac{413}{720}$.

P. 112, A. 125.

4. $\frac{36}{3}$, $\frac{18}{2}$.
5. $\frac{72}{8}$, $\frac{21}{3}$.

P. 113, A. 125.

6. $\frac{192}{16}$, $\frac{52}{4}$.
7. $\frac{75}{15}$, $\frac{75}{5}$.
8. $\frac{252}{18}$, $\frac{153}{9}$.
9. $\frac{375}{5}$, $\frac{252}{3}$.
10. $\frac{460}{4}$, $\frac{672}{8}$.
11. $\frac{774}{9}$, $\frac{584}{8}$.

P. 113, A. 126.

4. $\frac{923}{9}$.
5. $\frac{2364}{10}$.
6. $\frac{3015}{14}$.
7. $\frac{393}{8}$.
8. $\frac{697}{11}$.
9. $\frac{1235}{14}$.
10. $\frac{1557}{16}$.
11. $\frac{1682}{20}$.
12. $\frac{5037}{44}$.

P. 114.

1. { $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, & $\frac{42}{210}$, $\frac{35}{210}$, $\frac{30}{210}$.
2. { $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, & $\frac{126}{315}$, $\frac{135}{315}$, $\frac{140}{315}$.
3. { $\frac{3003}{8008}$, $\frac{5096}{8008}$, $\frac{4928}{8008}$, $\frac{5720}{8008}$. $\frac{825}{1320}$, $\frac{720}{1320}$, $\frac{924}{1320}$, $\frac{352}{1320}$.
4. $\frac{8}{12}$, $\frac{6}{12}$, $\frac{9}{12}$.
5. { $\frac{35035}{45045}$, $\frac{27027}{45045}$, $\frac{28610}{45045}$, $\frac{36855}{45045}$, $\frac{34650}{45045}$.

P. 115.

5. $\frac{32}{80}$, $\frac{30}{80}$, $\frac{25}{80}$.
6. $\frac{90}{120}$, $\frac{96}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.
7. $\frac{42}{150}$, $\frac{55}{150}$, $\frac{80}{150}$.
8. $\frac{32}{240}$, $\frac{75}{240}$, $\frac{132}{240}$.
9. $\frac{140}{180}$, $\frac{132}{180}$, $\frac{165}{180}$.
10. $\frac{27}{45}$, $\frac{10}{45}$, $\frac{30}{45}$, $\frac{21}{45}$.
11. $\frac{4}{54}$, $\frac{3}{54}$, $\frac{42}{54}$, $\frac{15}{54}$.
12. $\frac{175}{1120}$, $\frac{120}{1120}$, $\frac{616}{1120}$.

P. 117.

WRITTEN WORK.

1. $1\frac{7}{80}$.
2. $3\frac{31}{120}$.
3. $1\frac{9}{50}$.
4. $\frac{239}{240}$.
5. $2\frac{77}{180}$.
6. $1\frac{43}{45}$.
7. $1\frac{5}{27}$.
8. $\frac{911}{1120}$.
9. $1\frac{187}{1008}$.
10. $\frac{437}{630}$.
11. $11\frac{1}{8}$.

12. $14\frac{1}{28}$.
13. $19\frac{17}{36}$.
14. $14\frac{11}{180}$.
15. $14\frac{5}{9}$.
16. $15\frac{11}{14}$.
17. $17\frac{7}{12}$.
18. $18\frac{53}{90}$.
19. $8\frac{2}{15}$.
20. $12\frac{9}{40}$.

P. 118.

4. $\$21\frac{35}{72}$.
5. $\$42\frac{5}{16}$.
6. $19\frac{13}{16}$ lbs.
7. $20\frac{1}{16}$ yds.
8. $135\frac{7}{20}$ yds.
9. $5\frac{23}{40}$ tons.
10. $\$43\frac{3}{4}$.
11. $123\frac{263}{360}$.

P. 120.

13. $\frac{2}{165}$.
14. $2\frac{11}{45}$.
15. $3\frac{14}{175}$.
16. $1\frac{1}{4}$.
17. $6\frac{3}{5}$.
18. $1\frac{22}{35}$.

PROBLEMS.

2. $\$1\frac{1}{8}$.
3. $22\frac{5}{8}$ tons.
4. $4\frac{5}{18}$ yds.
5. $4\frac{97}{112}$.
6. James, $39\frac{19}{60}$ A.; Joseph, $182\frac{19}{30}$ A.; Daniel, $253\frac{13}{20}$ A.
7. $23\frac{1}{2}$ yds.

P. 121.

8. $\$1\frac{1}{8}$.

P. 122.

6. $8\frac{3}{9}$ cd., 10 cd.
7. \$10, \$20.
8. $13\frac{1}{2}$ bu.
9. $\$8\frac{8}{11}$, \$9.
10. $20\frac{4}{5}$ cts., 26 cts.
11. $\$58\frac{1}{2}$, $\$81\frac{9}{16}$.
12. $50\frac{1}{7}$ bbls., $66\frac{6}{7}$ bbls

P. 123.

5. $20\frac{5}{8}$.
6. $30\frac{5}{6}$.
7. $42\frac{4}{9}$.
8. 315.
9. $55\frac{5}{7}$.
10. $50\frac{3}{4}$.
11. $399\frac{3}{7}$.
12. $565\frac{2}{5}$.

P. 124.

14. $636\frac{1}{2}$.
15. 114.
16. 1344.
17. 4758.
9. $\frac{9}{14}$.
10. $\frac{14}{51}$.
11. $2\frac{2}{9}$.
12. 48.

P. 125.

13. $\frac{63}{80}$.
14. $\frac{3}{4}$.
15. $\frac{45}{272}$.
16. $\frac{39}{350}$.
17. $\frac{1}{6}$.
18. $15\frac{3}{7}$.
19. $\frac{17}{18}$.
20. $4\frac{1}{6}$.
21. $858\frac{3}{14}$.
22. $20\frac{4}{7}$.
23. $2\frac{1}{3}$.
24. $\frac{3}{28}$.
25. $75\frac{3}{5}$.
26. $2\frac{131}{299}$.
29. 15.
30. $18\frac{20}{27}$.
31. 1,090.
32. $3,000\frac{2}{3}$.

P. 126.

1. $\$3\frac{3}{4}$.
2. $\$\frac{5}{12}$.
3. $\$6.56\frac{1}{4}$.
4. \$547.50.
5. $\$10.46\frac{7}{8}$.
6. $\$4.38\frac{3}{4}$.
7. $\$226\frac{7}{8}$.
8. $\$31.71\frac{7}{8}$.
9. $\$78\frac{1}{8}$.
10. $51\frac{2}{3}$.
11. \$90.
12. \$25.
13. 2 tons.
14. $\$\frac{7}{44}$.
15. $\$5.46\frac{5}{6}$.
16. $\$1\frac{31}{64}$.

P. 128.

1. $1\frac{8}{19}$.
2. $\frac{2}{177}$.
3. $\frac{1}{44}$.
4. $\frac{14}{1953}$.
5. $\frac{561}{19600}$.
6. $\frac{172}{5135}$.
7. $\frac{5}{24}$.
8. $\frac{9}{32}$.
9. $\$\frac{2}{9}$; \$1 ; $\$1\frac{13}{18}$.
10. $\frac{8}{15}$ lb.

P. 129.

1. $7\frac{8}{11}$.
2. 130.
3. $1333\frac{1}{3}$.
4. 30.
5. $1102\frac{1}{2}$.
6. $14\frac{62}{67}$.
7. $7\frac{1}{32}$.
8. $11\frac{16}{25}$.

P. 130.

WRITTEN WORK.

1. $5\frac{31}{40}$.
2. $\frac{27}{70}$.
3. $\frac{9}{16}$.
4. $14\frac{7}{12}$.
5. $\frac{9}{14}$.
6. $9\frac{13}{18}$.
7. $5\frac{23}{44}$.
8. $\frac{45}{64}$
9. $1\frac{37}{200}$.
10. $149\frac{1}{3}$.

P. 131.

4. $\frac{45}{64}$.
5. $\frac{7}{8}$.
6. $\frac{28}{17}$.
7. $\frac{97}{36}$.
8. $\frac{1341}{1050}$.
9. $\frac{2577}{311}$.
10. $\frac{2}{45}$.
11. $\frac{455}{3456}$.

P. 132.

1. $7\frac{5}{11}$ bbls.
2. $31\frac{5}{18}$ bags. $15\frac{23}{36}$ b.
3. $\$6\frac{11}{48}$.
4. $13\frac{5}{8}$ yds.
5. $\$3.61\frac{1}{32}$.
6. $197\frac{3}{4}$ lbs.

P. 133.

7. $\$56\frac{1}{4}$.
8. $76\frac{1}{5}$ da.
9. $\$20\frac{1}{8}$.
10. $\$24\frac{15}{16}$.

P. 139.

1. .5601.
2. 1.0193.
3. .6127.
4. 13.326.

P. 140.

5. 37.2863.
6. 15.2646.
7. 466.0942.
8. 67.8155.
9. 2.9775.
10. 22.359.
11. 433.7279.
12. 163.763.
13. 449.881.
14. 85.855.
15. 57.32.
16. 11.9629.

PROBLEMS.

3. \$675.445.
4. \$10,093.144.

P. 142.

9. 16.439.
10. 5.318.
11. 9.0855.
12. 1.7866.
13. 0.444.
14. 2.108.
15. 3.27.
16. 94.25.
17. 232.694.
18. 1,313.7638.
19. 7.9437.
20. 63.63.

PROBLEMS.

3. 2,598.96.
4. 54.4188.
5. 574.755.

P. 144.

1. 15.04.
2. 2.544.
3. 40.02.
4. 1.0812.
5. 2.6325.
6. 39.442.
7. .0036.
8. 7.488.
9. 215.7.
10. 25.3825.
11. 1.090824.
12. 7.3926.
13. .27846.
14. .01716.
15. 172.86:
16. 24.75.
17. 13.52.
18. 568.944.
19. 125.6334.
20. 698.6395.
21. 3152.9696.
22. 7.8.
23. 842.
24. 473.
25. 650.

P. 145.

PROBLEMS.

1. \$15.05.
2. \$7.258.
3. \$134.075.
4. 22.932 mi.; 38.5875 mi
5. 1491.655.; 1638.175.
6. \$211.736.
7. \$16.936.
8. 391.82.; 462.275.
9. \$160.312.; \$218.025.
10. \$1.1509.; \$2.512.
11. 132.957 mi.
12. \$1.912.; \$4.556.

P. 147.

1. 2.96.
2. 14.96.
3. .016.
4. 1.6125.
5. 800.
6. 12.4.
7. 632.8.
8. .075.
9. .25.
10. .33.
11. 2000.
12. 56950.
13. 3.741.
14. 3.416.
15. .8437.
16. .8386.

P. 148.

1. $\frac{32}{1000}$.
2. $3\frac{112}{1000}$.
3. $7\frac{2}{1000}$.
4. 70, $\frac{3}{100}$.
5. $\frac{12}{100000}$.

P. 149.

1. .875.
2. .6875.
3. .6.
4. .44.
5. .714+.
6. .782+.
7. .85.
8. .818+.
9. .636+.
10. .692+.
11. .235+.
12. 6.1875.
13. 18.153+.
14. 10.363+.
15. .214+.
16. .882+.

PROBLEMS.

1. \$1.25, \$2.50.
2. \$5.50, \$13.75.
3. 9.4 mi., 18.8 mi.

P. 150.

4. \$0.375, \$0.75, \$1.875.
5. 20.79 A.; 21.415 A.
6. 92.67 mi.
7. \$1.45.
8. 161.21 mi.
9. 11.5 cts.; 42.55 cts.
10. 5 hrs.

P. 154.

5. \$120.992.
6. \$194.725.
7. \$77.073.
8. \$47.838.
9. \$12.73, \$4.195.
10. \$5.889.
11. \$17.46.
12. \$34.53.
13. \$25.19.
14. \$8.685.
15. \$4.49.
16. \$81.35.
17. \$42.75.
18. \$164.125.
19. \$2,396.925.
20. \$1,029.07.
21. \$13.14.
22. \$87.145.
23. \$26.
24. \$33.33.

P. 156.

1. 100 s., 160 s.
2. 216d., 132d.
3. £2, £3.
4. \$48.665; \$38.932.
5. 5 sov., 6 sov.
6. 600 ct.
7. 50 dc.
8. 2.5 fr.
9. \$27.02.
10. 103.626 + fr.
11. \$19.466.
12. \$14.5995.
13. \$1255.557.
14. 241.25.
15. 15.937 +.

P. 160.

1. 3 m.
2. 8 m.
3. 12 m.
4. 1500 m.

P. 161.

5. 20,000 m.
6. 5,000,000 m.
7. 300,000,000 m.
8. 2 sq. m.
9. .3 sq. m.
10. .006 sq. m.
11. 1500 sq. m.
12. 270,000 sq. m.
13. 5 st.
14. 1.5 st.
15. 3 st.
16. 40 st.
17. 200 st.
18. 5 l.
19. 12 l.
20. 2 l.

21. 3 l.
22. 4 l.
23. .2 Dg.
24. 50,000 g.
25. 40 cg.
26. 6 Kg.
27. 500 g.
28. 5.656 Kg.

P. 162.

2. $27.50.
3. $93.75.
4. $5.18.
5. $12.187.
6. $40.006+.
7. $91.057+.

P. 163.

3. $0.555. $4.44.
4. $0.75, $2.25.
5. $0.625, $10.3125, $16.25
6. $4.16.
7. 125 lbs.
8. $43.125.

P. 164.

2. 117 yds., 256.41+ yds.
3. 39 bbls., 50.24+ bbls.
4. 17.25 wks.
5. 311 yds. 775.19+ yds.
6. 29 bbls., 100 bbls.
7. 137.5 lbs., 275 lbs.
8. 27 bu., 81 bu.

P. 165.

2. $10.137+.
3. $30.59.
4. $16.20.
5. $28.125.
6. $6.48.

P. 166.

2. $28.476.
3. $34.355.
4. $65.408.
5. $59.416.
6. $2.326+.
7. $88.04.
8. $44.155.

P. 167.

2. $\$13.33\frac{1}{3}$.
3. $21.75.
4. $29.40, $36.
5. $21, $56.
6. $24.
7. $57, $168.75.
8. $14.75, $\$7.37\frac{1}{2}$.

P. 168.

2. 30.
3. 176.
4. 108 lbs., 200 lbs.
5. 15 qts. 25 qts.
6. 135 bu.

P. 170.

1. $26.60.
2. $258.05.
3. $294 Dr.

P. 181.

10. 18 sc., 36 sc.
11. 12 pks.
12. 370 min.
13. 14 pts., 20 pts.
14. 180 d., 510 d.
15. 180°, 360°.

P. 182.

1. 805 dwt.
2. 9,484 grs.
3. 475 grs.
4. 2,260 grs.
5. 1,444 oz.
6. 13,079 lbs.
7. 1,228 hrs.
8. 6,762 min.
9. 1,670,400 sec.
10. 3,345 in.
11. 9,246 ft.
12. $6,130\frac{2}{25}$ in.
13. 32,300 li.
14. 10,962 sq. ft.
15. 35,840 A.
16. 200,074 sq. li.
17. 34,728 cu. in.
18. 316 pts.
19. 361 pts.

P. 183.

16. 16 hrs.
17. 8 min.
18. 40 doz.

P. 184.

2. 2 lbs. 16 dwts. 7 grs.
3. 3 lb. 7 oz. 16 dwts.
4. 3℥ 1ʒ 11 grs.
5. 3 qrs. 24 lbs. 10 oz.
6. 4 T. 8 cwt. 1 qr. 17 lbs.; 2 T. 4 cwt. 0 qr. 12 lbs.
7. 5 da. 7 hrs.
8. 4 wks. 15 hrs.; 2 wks. 12 hrs.
9. 21 yds. 2 ft. 6 in.; 43 yds. 2 ft.
10. 2 mi 257 rds.; 5 mi. 194 rds.
11. 79 yds. 2 ft. 7 in.; 15 yds. 2 ft. 11 in.
12. 7 bu. 3 pk. 7 qt.; 3 bu. 3 pk. $7\frac{1}{2}$ qt.
13. 11 bu.; 22 bu.
14. 105 gal.; 52 gal. 2 qts.
15. 266 gal. 2 qt. 1 pt.; 66 gal. 2 qt. 1 pt. 1 gi.
16. 2° 27′ 21″.; 1° 13′ 42″.
17. 6 cu. yds.
18. 25 cds.
19. 36 A.

P. 186.

1. £10 10s.
2. 7ʒ 2℈ 6 grs.
3. 18 cwt. 3 qr. 13 lb.
4. 14 wks. 4 hrs.
5. 12 yds. 8 in.
6. 76° 13′ 15″.

P. 187.

7. 13 T. 11 cwt. 3 qrs.
8. 27 oz. 5 dwts. 19 grs.
9. 24 gal. 1 qt. 1 pt.
10. 34 bu. 3 pks. 3 qts.

P. 189.

5. 124 T. 59 gal.
6. 14 yrs. 46 wks. 4 da. 20 hrs. 58 min. 54 sec.
8. 12 yrs. 3 mos. 24 da.
9. 21 yrs. 4 mos. 20 da.

P. 191.

6. 157 lbs. 2 oz. 14 dwt.
7. 135 cwt. 24 lbs.
8. 211 da. 2 hrs. 51 min.
9. 113° 18′ 31″.

P. 193.

1. 4 da. 3 hrs. 15 m.
2. 4 wks. 5 da. 17 hrs.
3. 3 bu. 2 pks. 3 qts.
4. 5 cwt. 2 qrs. 16 lbs.
5. 6 lbs. 3 oz. 8 dwts. $13\frac{1}{3}$ grs.
6. 1 cwt. $10\frac{3}{13}$ lbs.
7. 5.
8. 8.
9. 11.
10. 5.

P. 198.

2. 8.05 lbs., 1.75 lbs., 3.50 lbs.
3. $34.65, $27.28, $66.
4. 8.32 wks., 3.2 wks., 8 wks.
5. 1030 yds., 1410 yds.
6. 28.8 bu., 120 bu.

P. 199, A. 232.

7. $300, $1008, $940.80.
8. 87 ft., 500 ft., 600 ft.
9. 84 gals., 132.3 gals., 197.4 gals.
10. $36, $300.
11. $3.75, $30, $22.50.

P. 199, A. 233.

2. $405.
3. 707.6 yds.
4. 87.4 A.
5. $237.62.
6. 86.4 T.

P. 199, A. 234.

2. 32 rds.
3. $378.
4. 58.88 wks.
5. 616 yds.
6. $976.50.

P. 200, A. 235.

2. 7%. $1\frac{2}{3}$%.
3. 5%.
4. 20%.
5. 10%.
6. 1%, $1\frac{1}{99}$%.

P. 200, A. 236.

2. 118.4.

P. 201.

3. $401.78\frac{4}{7}$.
4. 3,300.
5. 125 pupils.

PROBLEMS.

1. 4,875.
2. $3,622.50.
3. 70 chickens.; 112 ducks.
4. $36\frac{2}{25}$ gal.
5. $13,000.

P. 202.

4. $74.28+.
5. $47.25.

P. 203.

6. $176.62+.
7. $342.75.
8. $17.50.
9. $25.
10. $1886.19.

P. 204.

2. $96.
3. $780.
4. $27.20.
5. $12.650.
6. $26\frac{2}{3}$ cts.
7. $815.50.

P. 206.

3. $420.
4. $292.82.
5. $148.75.
6. $91.
7. $140.40.
8. $255.
9. $588.
10. $294.
11. $178.
12. $617.10.
13. $1587.20.
14. $2100.40.
15. $2140.352+.
16. $1873.521+.

P. 207.

3. $9.
4. $9.33\frac{1}{3}$.

P. 208.

5. $5.
6. $6 40.
7. $\$13.82\frac{1}{2}$.
8. $156.26.
9. $217.09.
10. $10.26.
11. $143.
12. $8102.40.

P. 209.

1. $1.34+.
2. $3.048.
3. $2.
4. $0.751+.
5. $.75.
6. $1.822.
7. $0.675.
8. $0.68+.

P. 211.

3. $224.583+.
4. $136.125.
5. $1182.75
6. $836.92.
7. $1381.415.
8. $1418.75.

P. 213.

3. $1,102.08.
4. $1,331.52.

P. 214.

2. $515.316.

P. 216.

1. $438.59+.
2. $33.963.

P. 217.

1. $1,975.50.
2. $2,945.75.

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